

# Math Champion Training 2018

*“I’ve missed more than 9000 shots in my career. I’ve lost almost 300 games. 26 times, I’ve been trusted to take the game winning shot and missed. I’ve failed over and over and over again in my life. And that is why I succeed.”*

- Michael Jordan

**Problem 1.** What is the last digit of  $2^{2^{10}}$ ?

**Problem 2.** Suppose  $x_1 > y_1 > 0$ , and  $y_2 > x_2 > 0$ . What is the length of the shortest path from  $(x_1, y_1)$  to  $(x_2, y_2)$  that touches both the  $x$ -axis and the  $y$ -axis?

**Problem 3.** Suppose we arrange the numbers  $1, 2, \dots, 64$  on a standard 8 by 8 chess board, with one number in each square. We say the *discrepancy* of a particular arrangement is the maximum difference between any two adjacent squares (where we consider any two squares sharing a side or corner to be adjacent). Out of all possible arrangements of the numbers on the board, what is the smallest possible discrepancy?

**Problem 4.** Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

**Problem 5.** Suppose 5 points are on the surface of a sphere. Show that there is some closed hemisphere (i.e. we draw some plane through the center of the sphere cutting it in half, and a closed hemisphere is one of the halves plus the boundary) containing 4 of our 5 points.

**Problem 6.** Let  $S$  be a set of 10 integers. Show that there is a nonempty subset  $T \subseteq S$  such that the sum of the elements of  $T$  is divisible by 10.