Math Prize Competition!

Western Michigan University

October 27, 2018

Name (printed legibly): ______________________________

Wmich email address: ______________________________

Read all of the following before working on the contest problems:

- This contest is **closed book**. Discussing the problems with colleagues is **not** permitted during the contest. The use of a calculator is **not** allowed for this contest. The use of a cell phone is **not** allowed for this contest. A violation of these rules results in immediate disqualification.
- Show or explain all of your work. Please write clearly and completely. Unjustified answers are regarded as incorrect.
- Please keep your written answers brief; be clear and to the point. (Do not ramble.)
- The problems are not necessarily in order of difficulty. If you spend more than 5–10 minutes thinking about a problem with no progress, you should probably look at the next one to see if it's easier.
- This contest has 6 problems and each problem is worth 10 points. If you do not have every page, please inform a proctor.
- You have three hours to complete your work on these problems.
- If you need clarification on any problem, please ask a proctor.
- Good luck!

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	**60**	

Problem 1. For $k > 0$, let $I_k = 10^{k+2} + 64 = 10 \ldots 064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the largest value of k such that I_k is divisible by 2^k. What is the maximum value of $N(k)$?
Hint: The answer is not 6.

Problem 2. Which is bigger, $\log_2(3)$ or $\log_3(5)$?
Recall that $a = \log_b(x)$ is the real number such that $b^a = x$, where $b \neq 1$ and $x > 0$

Problem 3. We say that an integer n is *sweet* if $n = a^2 + b^3$ for some **nonnegative** integers a, b. Out of all the values $1, 2, \ldots$ 1,000,000, are more than half of them sweet?

Problem 4. Suppose each of the variables $x_1, x_2, \ldots, x_{2018}$ is either 0 or 1. Prove that

$$x_1 + x_2 + \ldots + x_{2018} - x_1x_2 - x_2x_3 - \ldots - x_{2017}x_{2018} - x_{2018}x_1 \leq 1009,$$

and also give a specific example of values for $x_1, \ldots x_{2018}$ such that equality occurs.

Problem 5. Consider four real numbers x, y, a, and b, satisfying $x + y = a + b$ and $x^2 + y^2 = a^2 + b^2$. Prove that $x^n + y^n = a^n + b^n$, for all $n \in \mathbb{N}$.

Problem 6. All five vertices of a pentagon lie on the same circle. All five angles in this pentagon are equal. Is it necessarily true that all five sides of the pentagon have equal length?