

Math Prize Competition!

Western Michigan University

October 1, 2016

Name (printed legibly): _____

Which email address: _____

Read all of the following before working on the contest problems:

- This contest is **closed book**. Discussing the problems with colleagues is **not** permitted during the contest. The use of a calculator is **not** allowed for this contest. The use of a cell phone is **not** allowed for this contest. A violation of these rules results in immediate disqualification.
- Show or explain all of your work. Please write clearly and completely. Unjustified answers are regarded as incorrect.
- Please keep your written answers brief; be clear and to the point. (Do not ramble.)
- The problems are not necessarily in order of difficulty. If you spend more than 5–10 minutes thinking about a problem with no progress, you should probably look at the next one to see if it's easier.
- This contest has 6 problems and each problem is worth 10 points. If you do not have every page, please inform a proctor.
- You have two hours to complete your work on these problems.
- If you need clarification on any problem, please ask a proctor.
- Good luck!

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

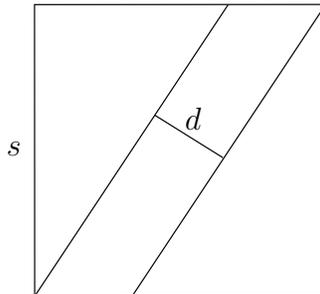
Problem 1. Professor X has a math class with 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

Problem 2. Compute the slope of the curve $y^5 + y^3 = x^3$ at the origin.

Problem 3. If $x^2 = 1 + zx^3$ and $y^2 = 1 - zy^3$ with $x, y > 0$, prove that $xy = 1 + z^2x^3y^3$.

Problem 4. Call sets A and B *almost disjoint* if they have at most one element in common. With n a fixed positive integer, how many members are in the the largest family of pairwise almost disjoint subsets from $\{1, 2, 3, \dots, n\}$?

Problem 5. In the picture below, the two parallel cuts divide the square into three pieces of equal area. The distance between the two parallel cuts is d . The square has length s . Find and prove a formula that expresses s as a function of d .



Problem 6. Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N , but by the end of the season, $S(N)$ was more than 80% of N . Was there necessarily a moment in between when $S(N)$ was exactly 80% of N ?