

WMU Math Club

Math Challenge Problem

October 2023

The Problem.

Assume that n and r are positive integers with $r \leq n$. Each of the $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ subsets of $\{1, 2, 3, \dots, n\}$ of cardinality r has a least element. Prove that the arithmetic mean of these least elements is

$$F(n, r) = \frac{n+1}{r+1}$$

for every choice of positive integers n and r with $r \leq n$.

Instructions.

1. Solve the problem.
2. Type your solution to the problem, preferably in $\text{\TeX}/\text{\LaTeX}$.
3. Email your solution to david.richter@wmich.edu with the phrase "October Challenge Problem" in the subject field before November 1, 2023.

More Information. If you submit the best solution, explained clearly and completely (and succinctly), then your solution will be posted on the WMU Math Club bulletin board next month, you will be recognized as the winner during the next meeting of the WMU Math Club, and you will receive a prize (probably a book). All undergraduate and graduate students may submit solutions. Do not submit more than one solution; every submission after your initial submission will be ignored. Please include your name in your write-up. Please make contact with Prof. David Richter if you have any questions.

Report from September 2023. Among the two submissions, neither was correct. The value of the limit was $\lim_{n \rightarrow \infty} x_n = (p+1)^{1/p}$.