WMU Math Club
Math Challenge Problem
March 2023

The Problem.

Suppose $a, b \in \mathbb{R}$ with $0 < a < b$. Evaluate the limit,

$$\lim_{n\to\infty} \frac{1}{n} \left[\int_a^b [\ln(1 + e^{nx})]^n \, dx \right]^{1/n}.$$

Instructions.

1. Solve the problem.

2. Type your solution to the problem, preferably in TEX/LATEX.

3. Email your solution to `david.richter@wmich.edu` with the phrase "March Math Challenge" in the subject field before April 1, 2023.

More Information. If you submit the best solution, explained clearly and completely (and succinctly), then your solution will be posted on the WMU Math Club bulletin board next month, you will be recognized as the winner during the next meeting of the WMU Math Club, and you will receive a prize (probably a book). All undergraduate and graduate students may submit solutions. Please include your name in your write-up. Please make contact with Prof. David Richter if you have any questions.

Report from Last Month. Alice Hoover (WMU) and Jason Shaye (KAMSC) both submitted correct solutions, and these were the only two submissions. $\sqrt{\mathbf{W}}$