WMU Math Club
Math Challenge Problem
January 2023

The Problem.

> For $x \in \mathbb{R}$, let $\lceil x \rceil$ denote the least integer not less than x (known as the "ceiling" of x). Prove that $\left\lceil \left(1 + \sqrt{3}\right)^{2n} \right\rceil$ is divisible by 2^{n+1} for every positive integer n.

Instructions.

1. Solve the problem.

2. Type your solution to the problem, preferably in TeX/LaTeX.

3. Email your solution to `david.richter@wmich.edu` with the phrase "January Math Challenge" in the subject field before February 1, 2023.

More Information. If you submit the best solution, explained clearly and completely (and succinctly), then your solution will be posted on the WMU Math Club bulletin board next month, you will be recognized as the winner during the next meeting of the WMU Math Club, and you will receive a prize (probably a book). All undergraduate and graduate students may submit solutions. Please include your name in your write-up. Please make contact with Prof. David Richter if you have any questions.

Report from Last Month. Andrew Bowling, Ritabrato Chatterjee, Darryl Jent, and Nick Vasenkov solved the problem from December 2022. The solution by Darryl Jent was judged the best by a graduate student and the solution by Ritabrato Chatterjee was judged the best by an undergraduate student. $\sqrt{\mathbf{W}}$