



DEPARTMENT OF MATHEMATICS

Math Club



## Michigan Epsilon Chapter of Pi Mu Epsilon

### Problem of the month: October 2019

#### A pebble game



Lily and Bharat play a game. They take turns (Lily goes first), and they start with 1000 pebbles on the table. On a player's turn, assuming there are  $N$  pebbles on the table, the player will remove one or more pebbles so that the new total is either  $N - 1$  or  $\lfloor \frac{N}{2} \rfloor$  (the player gets to choose which one). Whoever is able to remove the last pebble is the winner.

One of the players has a winning strategy. In other words, that player has a plan that will allow them to win no matter what the other player does. Which player has the winning strategy? Justify your answer!

**Note:**  $\lfloor x \rfloor$  is equal to  $x$  if  $x$  is an integer, and otherwise  $\lfloor x \rfloor$  is the largest integer that is less than  $x$ . So  $\lfloor 7 \rfloor = 7$  while  $\lfloor 2.4 \rfloor = 2$ .

Please turn in your solutions to Patrick Bennett, by noon on **Thursday October 31, 2019**. Strive for clarity, neatness and legibility! Solutions may be turned into the Math Dept office in **3319 Everett Tower**. Please include your name and email address. Electronic submissions may be sent to [patrick.bennett@wmich.edu](mailto:patrick.bennett@wmich.edu). If you are currently taking a math class, please include the instructor's name and the course number.

<http://www.wmich.edu/mathclub>