



DEPARTMENT OF MATHEMATICS

Math Club



Michigan Epsilon Chapter of Pi Mu Epsilon

Problem of the month: February 2020

Long sequences

$$(100, 55, 45, 10, 35)$$

In the sequence above, each term starting with the third term is the difference of the previous two. In other words, $a_i = a_{i-2} - a_{i-1}$ for each $i \geq 3$. The sequence ends when the next term would be negative (here the next term would be -25 so we stopped). Terms equal to 0 are allowed, but no negative terms.

Suppose you would like to start such a sequence with $(100, x, \dots)$ for some positive integer x . What is a value of x that will make your sequence as long as possible? Generalize your solution to solve the following problem: if you are given a value of y and your sequence starts (y, x, \dots) then find a value of x (depending on y) that will maximize the length of the sequence. Justify your answer!

Please turn in your solutions to Patrick Bennett, by noon on **Friday February 28**. Strive for clarity, neatness and legibility! Solutions may be turned into the Math Dept office in **3319 Everett Tower**. Please include your name and email address. Electronic submissions may be sent to patrick.bennett@wmich.edu. If you are currently taking a math class, please include the instructor's name and the course number.

<http://www.wmich.edu/mathclub>