Department of Mathematics

Math Club

# Michigan Epsilon Chapter of Pi Mu Epsilon

# Problem of the month: January 2020

## Roots and nonzero coefficients

Suppose we have a fifth degree polynomial

$$p(x) = (x - r_1)(x - r_2)(x - r_3)(x - r_4)(x - r_5)$$

where the roots $r_1, \ldots, r_5$ are **distinct integers**. If we expand $p(x)$, what is the smallest possible number of nonzero coefficients?

For example, if

$$p(x) = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$$

and we expand to get

$$p(x) = x^5 - 15x^4 + 85x^3 - 225x^2 + 274x - 120$$

we see that all six coefficients are nonzero.

However if instead we have

$$p(x) = x(x - 6)(x + 1)(x + 2)(x + 3) = x^5 - 25x^3 - 60x^2 - 36x$$

then we see only four nonzero coefficients. Can you choose distinct integers $r_1, \ldots, r_5$ so that $p(x)$ has only three nonzero coefficients? Can you get only two nonzero coefficients?

Please turn in your solutions to Patrick Bennett, by noon on **Friday January 31, 2020**. Strive for clarity, neatness and legibility! Solutions may be turned into the Math Dept office in **3319 Everett Tower**. Please include your name and email address. Electronic submissions may be sent to **patrick.bennett@wmich.edu**. If you are currently taking a math class, please include the instructor's name and the course number.

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