WESTERN MICHIGAN UNIVERSITY
Graduate College

Announcement of Doctoral Oral Examination

Candidate:	**Abdul R. M. Nasser**
For the degree of:	Doctor of Philosophy
Department:	Mathematics
Title:	**On Near Vector Space Cellular Automata**
Committee:	Dr. Clifton Ealy, Chair Dr. David Richter Dr. Elise de Doncker Dr. Jonathan D.H. Smith
Time/Place:	**Monday, May 13, 2024** **2 to 4 p.m.** **Virtual meeting**

Cellular Automata can be considered as examples of massively parallel machines. They are computational mathematical objects consisting of a grid of cells, each of which can exist in a finite number of states. These cells evolve over discrete time steps according to a set of predefined rules based on the states of neighboring cells. The notion of cellular automata was first introduced by Ulam and von Neumann and then popularized by John H. Conway in the 1970s with one of the most famous examples being The Game of Life.

This research builds on and generalizes the work of Tullio Ceccherini-Silberstein and Michel Coornaert, who in 2010 studied cellular automata over vector spaces inspired by the works of Mikhael Gromov regarding the endomorphisms of symbolic algebraic varieties. In this framework, the configuration space $\hom(G, V) = V^G$, where G is a group and V is a vector space

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over a field K, naturally forms a vector space over K. Cellular automata are endomorphisms of this vector space, which are continuous and invariant under the natural G action on V^G.

In this body of work, cellular automata whose alphabets are N^n, where N is a near-field or an Abraham Adrian Albert (A.A.A) division algebra are investigated. Near-fields, discovered by Leonard E. Dickson in 1905, generalize both fields and division rings. Hans Zassenhaus noted that finite near fields were "sharply 2-transitive groups". Since then, near-fields have been intensively studied in the guise of sharply 2-transitive groups. Specifically, focus is on the structure of near-linear cellular automata, which are cellular automata over groups whose alphabets are near vector spaces, N^n for N a near-field or A.A.A division algebra. These cellular automata exhibit near linearity with respect to the induced near space structure on the set of mappings $\hom(G, N^n)$.

The main results of this work consist of developing a near-linear analog for the Curtis-Hedlund Theorem, exploring a Garden of Eden-type theorem for near vector spaces, and lastly exploring the correlation between sofic groups and the property of near-linear surjunctivity.