

DEPARTMENT OF MATHEMATICS Math Club



Michigan Epsilon Chapter of Pi Mu Epsilon

Problem of the month: September 2019

Rook tours



You are a rook who lives on an $s \times t$ checkerboard. You would like to visit every square on the board exactly once, and you may start and finish on whatever (different) squares you like. You may only move horizontally or vertically, but you may go any distance in a single move (like a rook in chess). Unlike chess, you must alternate between vertical moves and horizontal moves. In other words if your previous move was vertical then your next move must be horizontal, and vice versa. Your first move may be either vertical or horizontal. A square is considered to be "visited" if you start there or if you arrive there at the end of a move. The squares that you "jump over" during your move are not considered to be visited during that move.

For example, the case where s = t = 2 is easy. You can start in the top left square, then move to the top right, then the bottom right, and finally the bottom left.

Problem: For what values of s and t is it possible to visit every square exactly once? Justify your answer (meaning that you must justify which values of s and t are possible by describing how to visit every square, and also show that all other cases are impossible).

Please turn in your solutions to Patrick Bennett, by noon on Monday September 30, 2019. Strive for clarity, neatness and legibility! Solutions may be turned into the Math Dept office in 3319 Everett Tower. Please include your name and email address. Electronic submissions may be sent to patrick.bennett@wmich.edu. If you are currently taking a math class, please include the instructor's name and the course number.

http://www.wmich.edu/mathclub