Do FIVE of the following seven problems

3. Let $X = \{a, b\}$ with the Sierpiński topology $\{\varnothing, \{a\}, X\}$.
   a. Is X connected?
   b. Is X path connected? Be careful.

4. Let A be a compact subset of a Hausdorff space X. Show if $x \notin A$ then there are disjoint open subsets U and V such that $x \in U$ and $A \subseteq V$.

5. Let X be a topological space and $D = \{(x, x) \mid x \in X\} \subseteq X \times X$ be the diagonal.
   a. Show X is homeomorphic to D.
   b. Show if X is Hausdorff, then D is closed.

6. Show that the finite complement (or cofinite) topology is a topology.

7. Let X be a topological space and A, B be two subsets of X. Prove

$$\operatorname{int}(A) \cap \operatorname{int}(B) = \operatorname{int}(A \cap B).$$

8. Let X be a normal space with at least two points. Show X is uncountable.

9. a. Show the Möbius strip is homotopy equivalent to the annulus

$$A = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 4\}.$$

   b. Show the Möbius strip is not homeomorphic to the annulus A.