There's Music in the Air

Sound and Sinusoids

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Table of Contents

Why and what the students learn :

Because many high school students have instruments or at least an interest in music, this can be used as a hook to interest even some of the most challenging students. They are fascinated during the data collection process and amazed at **the sinusoidal nature of a note played on their instrument, or from them humming into the microphone.**

During this activity the students should learn or be exposed to:

a) the difference between "Western" style music and "Eastern" style.

b) what three notes comprise the simplest major chords.

c) what the frequency, period, amplitude, and sinusoidal axis of a sinusoid means in the context of a musical note or sound and compute them.

d) how to write the equation of a cosine wave from the ordered pairs (time, displacement) obtained using a Calculator-Based Laboratory (CBL2™).

Type of student who has done this activity successfully:

I have used various forms of this activity with **all levels of high school students**. This activity has been done successfully with Advanced Placement Physics students (mostly juniors and seniors), and also our more math challenged students who are often placed in a course called Math Modeling (mostly 10th and 11th graders). I have used it with Advanced Algebra students (mostly sophomores and juniors) and with Math Analysis students (mostly juniors and seniors). Colleagues have done this in regular physics courses and also with their own math modeling students.

When I did this activity with math challenged students, we did several hands-on activities first. First we collected data from rolling a bike tire and measured the height of the valve stem and the distance the tire had rolled along the table. They graphed these ordered pairs by hand and developed scatterplots that were sinusoidal in nature.

We then all shared ordered pairs collected using a "Calculator Based Ranger" (CBR™). motion detector. A can was swung to obtain data (time, distance from the CBR™. This one set of data was put on all the calculators using a Navigator™ System. This is the most efficient way of data transfer but if you don't have access to a Navigator System, data can be transferred using the calculator-to-calculator link cable, which comes with all models of TI-84 and TI-83 calculators. With math challenged students these type of warm-up activites need to be done, and this actual activity needs to be done with at least 4 sets of data, before the students master getting the equation of the sinusoid associated with a note played or hummed.

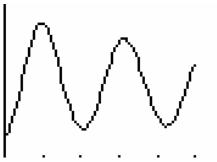
With more advanced students such as my math analysis students and physics students, you can just talk about natural phenomenon which is sinusoidal and the basic

characteristics of a sinusoid, and they are actually ready to collect this data, and find the sinusoid, especially when allowed to use the included "cheat sheet." In the case of these advanced students, you may not even want to give them the cheat sheet. This activity can be done in two normal class periods with advanced students. With students of lower ability you probably need about two weeks to do some introductory work and the activity.

Equipment necessary and/or desired:

For the tire activity, used as a warm-up to this activity, I used bicycle tires from student bikes, long quilting tape measures (bought at sewing stores), taped to the tables, and meter sticks. We then graphed their ordered pairs on normal graph paper and also using the STAT PLOT feature of their graphing calculators (TI-84's).

For swinging the softballs, I bought plant hangers for my dropped ceiling, and put picture hanger screws into the softballs. For this activity I used the CBR™ motion detectors available at the Texas Instrument web site. The data was instantly transferred to their graphing calculator, linked to the CBR™.



For this lab, you will need CBL2's™ (available at the TI website) and a good set of graphing calculators which are compatible with the CBL2's™. I would recommend TI-84 Plus Silvers, which are available at most office supply centers and large discount stores. Lower level models will also work but silvers have much more memory for the large data sets you will use, and have much faster processors. I have done this activity with the students in groups of three (by far my favorite group size), and I have done it individually. It works well both ways. I would recommend at least pairs working together on any warm-up activities you may do, and maybe for the first set of sound data they work with.

You will also need to purchase Vernier Microphones (order code MCA-BTA or MCA-CBL) available at the Vernier website (www.vernier.com). **http://www.vernier.com/probes/mca-bta.html**

If you have a Navigator™ System available in your school, this makes data distribution much easier, real-time assessment possible, and testing on this material very easy. This activity can be done without this system however.

Students brought in their guitars, bought one dollar flutes at the local dollar stores, brought trumpets, saxophones, violins, and other instruments. I always start by

showing the students that they can hum (works better than singing) a sinusoid too. I also used tuning forks and had they students determine the frequency of the fork.

Prerequisite knowledge and/or activities:

I've mentioned some of this in the section above called "Type of student who has done this activity successfully ." Also students at least need a working knowledge of a graphing calculator of some type – hopefully in the TI-82, TI-83, or TI-84 family, I have had students do this with TI-85's and TI-86's too. They students should have passed at least a beginning Algebra I course.

National Council of Teachers of Mathematics
Principles and Standards for School Mathematics
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"Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." *Pg. 24*

"Students can learn more mathematics more deeply with the appropriate use of technology." (Dunham and Dick 1994 etc.) pg. 25

"Student's engagement with, and ownership of, abstract mathematical ideas can be fostered through technology." Pg. 25

Data Analysis and Probability Standard:
Select and use appropriate statistical methods to analyze data.

- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients using technological tools.
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

Representation Standard:
"Use representations to model and interpret physical, social, and mathematical phenomena."

"Students are ready in high school to see similarity in the underlying structure of mathematical structure of mathematical objects that appear contextually different but whose representations look quite similar.

Connections Standard:
Recognize and apply mathematics in contexts outside of mathematics.

"As students' knowledge of mathematics, their ability to use a wide range of mathematical representations, and their access to sophisticated technology and software increase, the connections they make with other academic disciplines, especially the sciences and social sciences, give them greater mathematical power.

MICHIGAN'S STATE STANDARDS AND BENCHMARKS:

MAT.I.1.HS.3 – Study and employ mathematical models of patterns to make inferences, predictions and decisions.

MAT.I.1.HS.4 – Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations of relations.

MAT. III.1.HS.1 – Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.

MAT.III.2.HS.1 – Critically read data from tables, charts, or graphs and explain the source of the data and what the data represent.

The Activity:

Actual Lab Day:

Connect the CBL2's™ to the calculators using the short black cord which comes with every CBL2™. Plug the Vernier Microphones into Channel 1 on the left side of the CBL2's™.

Place the TI calculator in LINK mode by pressing the 2nd function and the Link key.
Use the right arrow to select "RECEIVE" on the calculator.

Press the three top buttons on the CBL2's™ in order from right to left:
START/STOP
QUICK SETUP
TRANSFER

The software you need for your data collection is automatically put into your calculator. The screen on your calculator will tell you when this is finished. It takes a couple minutes.

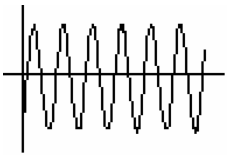
Now push the "APPS" button on your calculator, and go down to "DataMate."
When you press enter, the calculator will automatically check sensors and find out that you have a microphone plugged in and will automatically calibrate and zero out your equipment.

You can actually start collecting data at this time, or you can look through the menus.

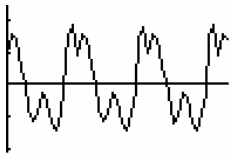
If you push "2)START" it will collect your first set of data. Get the microphone very close to the instrument, or the person humming and push start. Remember that a guitar tunes to 440, which means 440 waves per second. Your equipment will actually only collect data for about half a second using the default settings loaded automatically into your calculator. It will say "sampling" for longer than that so keep humming. =)

It will automatically graph your note. If you were not successful the first time in getting something, which looks like a sinusoid or at least a periodic function, you can just push "START" over and over until you get a good function.

If you like the graph you have, then have them copy this graph down including the labels. When you have this graph sketched, push "enter." Then push 6:QUIT and [Enter]. It takes the calculator about 15 seconds to leave this mode and return to the home screen. Some examples I have collected look like:



A Hummmmm



The saxaphone on the cover

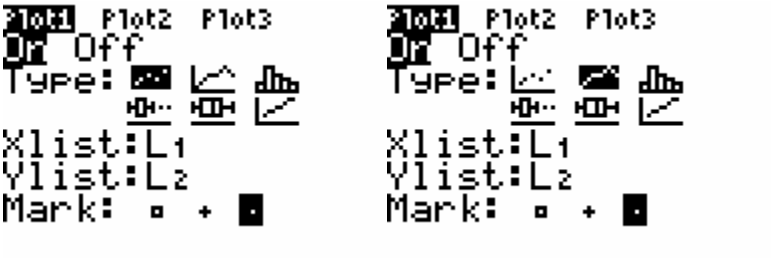
This data is now in your STAT lists. To view it, press the STAT key, and then 1: EDIT

You should have 201 ordered pairs stored. You now need to make a scatterplot of this data by pressing the 2nd function and going into the STAT PLOT settings on the "y=" key.

Press enter on Plot 1
You can find the L_1 on your calculator as a second function on your "1" key.
Set your menu up as the example below on the left.
Press ZOOM and ZoomStat which is ZOOM 9 (on top row of buttons).

You should automatically have a window that works well for the data. If you have an advanced class, have them figure out the window without this feature. At least look at the window that the calculator has selected and talk about the domain and range.

IF the 201 data points seem to not be sinusoidal or they can't seem to see a periodic function like they had immediately after collecting the data, use the settings as you see them on the right below. This connects the data points in time sequence.



When I want to help the students get started I now show them a graph on the overhead calculator, and we all decide what the ordered pair is for the highest point to the right of the y-axis. With an advanced class call this a "relative maximum" point. For a lower level class you might call this a "peak" as it is on the "Cheat Sheet" you will find further on in this report. Also, decide on the ordered pairs for the second peak to the right of the y-axis and the ordered pair for the valley (relative minimum) point between the two peaks.

With this information, they are ready to find the period for the function, the frequency, the "B" value needed in their equation, and they are also ready to find the other three values needed in the equation. They really can take it from here on their own, even the lower ability students. All they need is encouragement.

After they have found the equation of the periodic function, they should put this equation into their y-menu, and if the equation is correct, it will run nicely through their scatterplot.

I would suggest you now evaluate the fit on their calculator. Make suggestions as to getting a better fit if necessary, but otherwise the students must show me their calculator fit and that is at least half their grade. They then turn in the worksheet (included) showing their calculations.

To test them, I give them all sets of data I have collected, and they do the same activity alone, with the same requirements for showing me how their equation fits through the data, and turning in a worksheet with their calculations.

The much loved "Cheat Sheet"
for
Periodic Function Lab Activities

Review of terms:

Period : The length of one complete cycle on the X-axis.
For most applications this is a measure of time.

Frequency: The reciprocal of period or the number of cycles that occur per second.

$$\text{Freq} = \frac{1}{\text{Period}} \quad \text{so} \quad \text{Period} = \frac{1}{\text{Freq}}$$

$$\boxed{y = D + A\cos(B(x - C))}$$

D: Axis of the wave (Axis of oscillation): The center of the wave. It is a measure of how much the wave has been shifted up or down. **Calculated using the Y-axis**

$$D = \frac{\max + \min}{2}$$

A: Amplitude: the stretch up and down of the function. **It is measured on the Y-axis.**

$$A = \frac{\max - \min}{2}$$

Period: The length of one cycle on the X-axis. How long is one wave?

Period = peak – previous peak or bottom - previous bottom

B: When graphing in degrees $B = \frac{360\,\text{degrees}}{\text{period}}$ When graphing in radians $B = \frac{2\pi \text{ radians}}{\text{period}}$

B controls the graph's period but it is NOT the period.

C: The shift of the graph to the left or right. When writing COSINE equations (this is NOT true for sine), the C is the distance on the x-axis, from the y-axis to the first peak to the right of the y-axis.

Name _____ Block _____

Write the name of the data here!!! _____

Show what numbers you used to get the Period _____

Show what numbers you used to get A _____

Show what numbers you used to get B _____

Show the numbers you used to get D _____

Show the numbers you used to get C _____

Final Equation _____

Frequency of your note _____

Name _____ Block _____

Write the name of your data sets here ! _____

Show what numbers you used to get the Period _____

Show what numbers you used to get A _____

Show what numbers you used to get B _____

Show the numbers you used to get D _____

Show the numbers you used to get C _____

Final Equation _____

Frequency of your note _____

Name _____ Block ____

ALL students must visit the following two sites and write about them:

1)
Go to:

http://www.scroom.com/mus_lessons/tot.5.html#double_c5

What did you find at this site? _____
Go down to **II. Major Keys**

If you are playing guitar in the key of "G", the most likely two other chords (the fourth and fifth) you would play, are C, which precedes G on the circle, and D, which follows G in the circle, so G, C and D are the likely three chords used the most in a simple major song.

If you are playing in the key of A, what notes are the 4^{th} and 5^{th} ? _____

When playing in D, what notes are the 4^{th} and 5^{th} ? _____

Describe what this site does, in general _____

_____

_____

_____

2)
Go to:
http://www.jhu.edu/~signals/listen-new/listen-newindex.htm

Under "Introduction" At least read the last part – the part about Western Music.
Then fill in the following and follow the directions below that.

For Western music, an octave is divided into _____*notes* that are equally spaced on a _____ scale.
The ordering of notes in the octave beginning at _____ *Hertz* is shown…

Click on A 220 Hz
Click on B 247 Hz
Click on C 277
Click on D 294
Click on D# 311
Click on E 330
Click on F# 370
Click on G 392
Click on G# 415
Click on A 440

What do you notice about the wave as you click on these one-after-another?

_____

Attempt to write the first few notes of a song you know…

Song _____ Notes _____

3) Go down to the next activity “additional Topics”

Click on “sine” and sketch it here: _____

Click on “sawtooth” and sketch it here: _____

Click on “oboe” and sketch it here: _____

Click on ”clarinet” and sketch it here: _____

4) go down to the next activity “effect of Phase”

Click on “play” and describe what happened.

_____

_____

5) Go down to “Harmonic Contribution” What did it do?

_____

_____

6) Go down to “Filtering White Noise” What is “white noise?” _____

_____

What Hz cannot be heard by most humans? _____

Compare and Contrast Western vs. Eastern Music

Name _____

Write a compare and contrast paper (about **500** words long) comparing Western style music and Eastern style music **and** share this with the class in an **oral** report. Below is an example of what I have in mind BUT this is a direct cut and paste. Don't do this unless you can explain what you have written. In other words, your written and oral report should be:

A) material you understand
B) is in your own words

The written paper needs to be turned in AFTER you do the 3-5 minute oral report.

These are NOT my own words and I think that is obvious. This is an example of pertinent information I found on the internet. There are other differences too. Explore. Extra credit is possible if you can DEMONSTRATE. =)

"In typical Western music, there are 12 possible notes to choose from: A, A Sharp (also known as B Flat), B, C, C Sharp (also known as D Flat), D, D Sharp (also known as E Flat), E, F, F Sharp (also known as G Flat), G, and G Sharp (also known as A Flat). But only 7 of these notes are normally used in a given song. The 7 notes selected as the basis for a given song comprise a scale, and there are certain specific rules regarding which 7 notes can be used in a certain scale. For example, a typical Western song in the key of D Major will use only the notes D, E, F Sharp, G, A, B, and C Sharp. The scale also defines certain relationships between the notes for determining which notes the chords will consist of, and which chords will serve as the basic building blocks for a given song."

"In Arabic and Turkish music, the maqam utilizes only selected notes from the full range of possible notes available, so in that respect it resembles the Western scale. But the maqam goes farther in its influence on the resulting music. It also consists of a melody scrap based on certain key notes from that scale and a certain tendency of movement. So a song written in a certain maqam must not only use the particular notes in that maqam, but it must also incorporate the melody scrap for that maqam into the melody line of the song."

25 Points : Written report, in your own words, turned in _____

Approximately 500 words long _____

Oral : Talked about Western Music. _____
Included material about frequencies.
Oral : Talked about Eastern Music. _____
Included material about frequencies.
Oral: Compared Western to Eastern. _____

Oral report was 3-5 minutes long _____

Post Notes concerning this lab:

Between the time that I used this material with students and the time I put it in it's final form, one of the web addresses changed and I had to find another web address that worked with the circle of fifths. That was not very difficult, but be aware that some addresses go stale, or are replaced. Every time I bring students to the computer lab to work on an assignment such as those in this activity I check two things.

First I check to make sure web addresses are actually still working.

Second I check if the computer lab still has "Applets Enabled." This is something the person in charge of your lab probably has to do, because security on school computers usually won't allow you to change these settings. Also, check that the sound can be turned on, or have students bring headphones. It is better if they can hear the different notes.

The circle of fifths information helps to explain the "sound" that much Western Music has, because it is based on a 1-4-5 chord arrangement. I include it because so many students are budding guitar players and this gets them immediately involved.