

Going Off on a Tangent

Math and Global Positioning Systems (GPS) Activities

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Going off on a Tangent

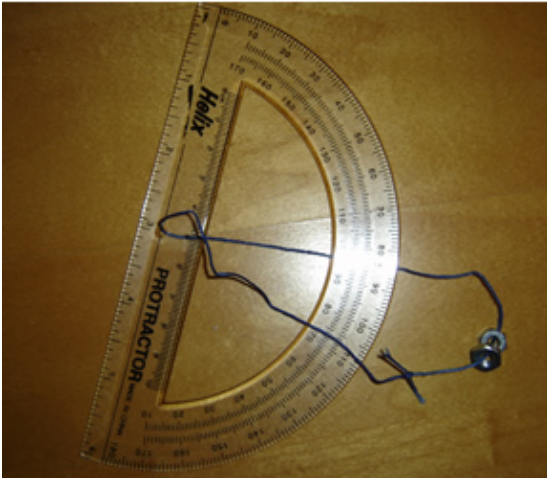
Teacher Instructions and Information.

Goal: Students will indirectly measure heights of objects using a sighting tube, tape measure and trigonometry.

Prior knowledge: Students should have had experience with working with trigonometric functions. In addition, they should have worked with protractors and tape measures and they should know the basics of accurate measurement.

Needed materials

- Student handout. (last page)
- **Protractor, string, and weights to make a transit level.** I have found that it works best when you make a giant loop of string so that you get a full range of angle measures.



- **Tape measures** (at least two per class. One per group would be ideal) Depending on the size of location a construction tape measure could also be used. However, the 100 ft tape measure allows students to get a wide range of angles of elevations.



Teacher Instructions

Tell the students to form groups of 3-4 people. Roles in the group include
2 people to work the tape measure
1 person to work the homemade transit level (protractor, string and weights)
1 person to record data collected

Teacher Instructions Continued

When collecting data it will be easiest to use round numbers for feet (that can use ft/in but it will make calculations more complex)

Have the students get one transit level one tape measure, and one worksheet before heading outside.

Question

If I look up 30° , what will be the reading on the transit level? (answer: 60—The string hangs at the 90° mark when you have a 0° angle of elevation.)

Question

What will need to be done with the angle that you measure on the transit level? (Answer—Subtract it from 90°)

There is space for students to make more than one measurement. Depending on the level your students are at you can use as many measurements as you would like to with your students. Tell the students that they will be graded on accuracy. The students need to learn to take their time with the measurements

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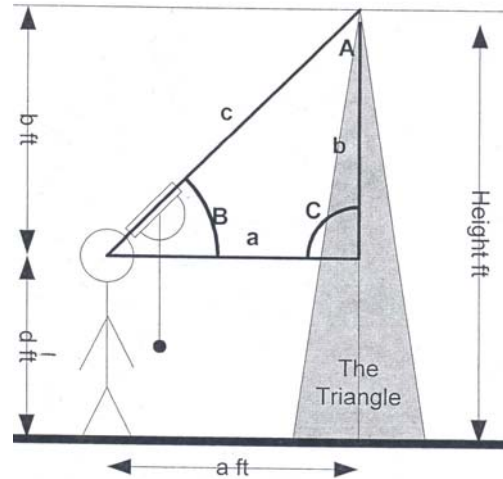
Often, you need to measure indirectly, because it is not practical or possible to measure directly. (Heights of mountains, heights of buildings without the building plans.). A few simple measurements will allow you to measure indirectly.

Needed materials:

- This handout.
- Protractor, string, straw, washer that make a transit level.
- Tape measure

Data collection:

1. Measure the height of the flagpole.
 - a. Use the tape measure to measure your distance from the flagpole.
 - b. Use the sighting tube to measure the angle of elevation.
 - c. Change position and repeat a maximum of 5 times.
 - d. Record the data in table below.
2. Measure the height of the auditorium using the above steps.



Calculations

Use trigonometry to find the height of the flagpole and the auditorium for each measurement, and then calculate the average of each. Finally, add your height to the average.

Height of Flagpole

Dist (ft.)	Angle (°)	Height (ft.)
Average Height		
Avg. + your height		

Height of Auditorium

Dist (ft.)	Angle (°)	Height (ft.)
Average Height		
Avg. + your height		

Questions

1. Explain what calculation you used to find the height of the objects. Suppose the sighting tube has a laser range finder (the ability to find distance). Describe the setup that you could use to find the height of the flagpole or auditorium.
2. What is the direct distance from the eyes of the angle measurer to the top of each object?
3. Why do you need to add the height of the angle measurer to the height you calculated?
4. Does it matter if the person's height is added to the average or each individual calculation? Explain.

Going off on a Tangent. Teacher Instructions.

Goal: To have students learn how to measure height indirectly using trigonometry.

$\sin x = \frac{o}{h}$, $\cos x = \frac{a}{h}$, $\tan x = \frac{o}{a}$ for a right triangle where x is the measure of a non-right angle and o is the opposite side, a is the adjacent side, and h is the hypotenuse.

Needed materials

- Student handout.
- Protractor, string, straw, washer to make a sighting tube. (enough for one for each group)
- Tape measure (at least two per class would be ideal)

Goal: Students will indirectly measure heights of objects using sighting tube and tape measure.

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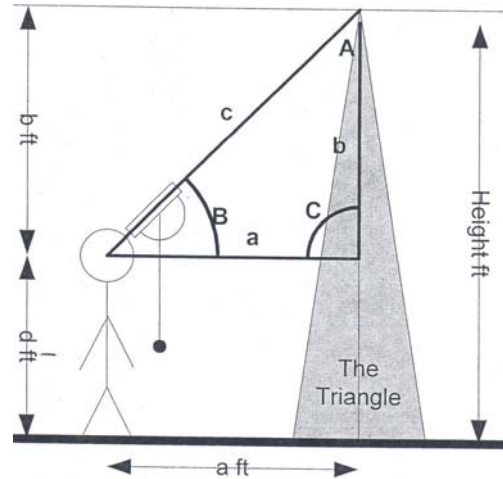
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 - c. Change position and repeat a total of 5 times.
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Calculations

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Dist (ft.)	Angle (°)	Height (ft.)
Average Height		
Avg. + your height		

Height of Auditorium

Dist (ft.)	Angle (°)	Height (ft.)
Average Height		
Avg. + your height		

Questions

5. Explain what calculation you used to find the height of the objects. Suppose the sighting tube has a laser range finder (the ability to find distance). Describe the setup that you could use to find the height of the flagpole or auditorium.
6. Why do you need to add your height to the height you calculated?

Where you at? It sounds good on paper. Teacher Instructions.

Goal: Use indirect measurement to locate a position on paper using trigonometry, and a system or two equations (advanced algebra) or matrix algebra using an over determined system of equations (pre calculus).

Needed materials.

1. Student worksheet.
2. Protractor.
3. Straight edge.

Review

Review solving equations, using protractors to measure angles, and definition of slope.

Advanced Algebra Mathematics

In advanced algebra, students have worked with systems of equations, simple matrices, and are currently learning trigonometry. Students have at their disposal several ways of solving systems of equations: substitution, elimination, matrices, and graphing on calculators.

$$y = \tan \theta_1(x - x_1) + y_1$$

$$y = \tan \theta_2(x - x_2) + y_2$$

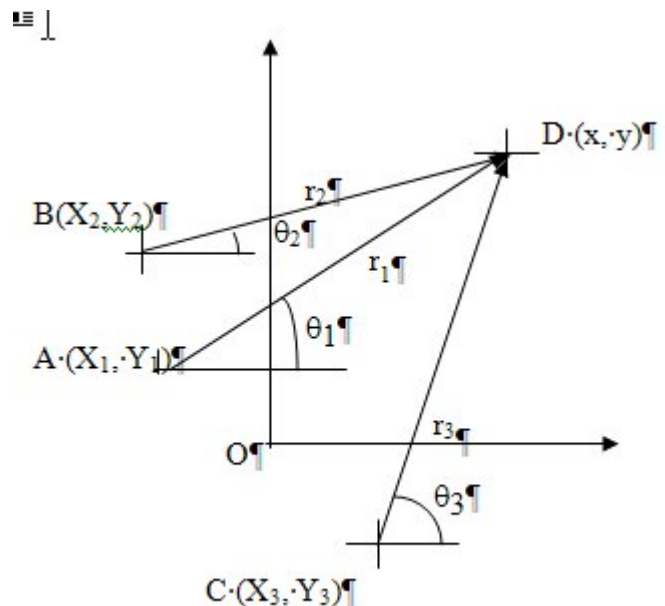
Pre Calculus Mathematics

In pre calculus, students will study matrices more in-depth and be able to do more advanced calculations using matrices. Namely using over determined systems of equations to evaluate to a greater precision, the unknown position. Students will need to know how to transpose a matrix, calculate inverses and set equations up into a collection of matrices to evaluate position.

$$\begin{pmatrix} 1 & 0 & \cos \theta_1 & 0 & 0 \\ 0 & 1 & \sin \theta_1 & 0 & 0 \\ 1 & 0 & 0 & \cos \theta_2 & 0 \\ 0 & 1 & 0 & \sin \theta_2 & 0 \\ 1 & 0 & 0 & 0 & \cos \theta_3 \\ 0 & 1 & 0 & 0 & \sin \theta_3 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \end{pmatrix}$$

$$P \times U = C$$

Using $[(P^T P)^{-1} P^T]^{-1} C$, the students will evaluate matrix U. With the ease of calculator setup, students can use as many points as they would like, but a reasonable number of points would be 3 to 5 points.

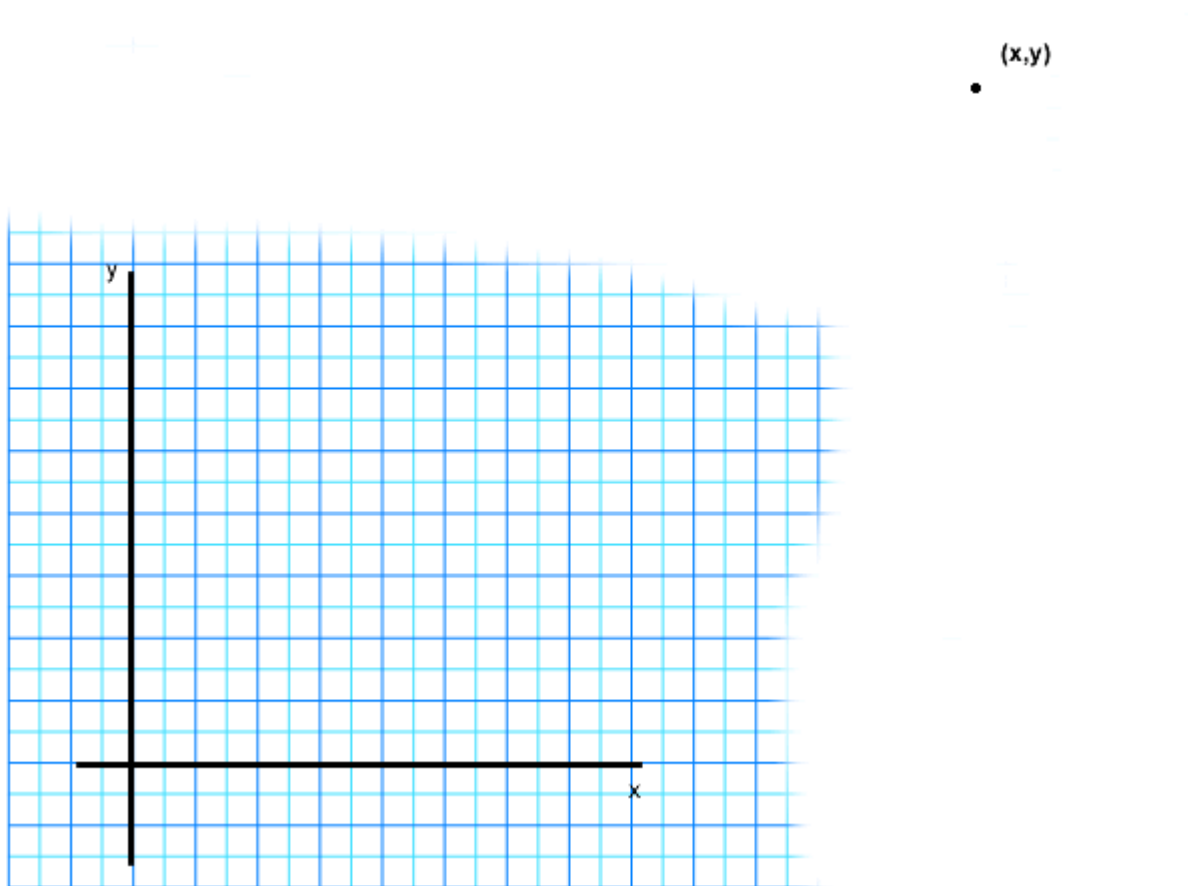


Where you at? [in brackets for pre-calculus instructions]

Trigonometry can be used to accurately assess the position of an object even when it is not directly accessible.

Below, a piece of graph paper has been ruined leaving only an axis system and an unknown coordinate point that is off of the grid.

1. Find 2 [3 to 5] different coordinates on the grid below.
2. Find the angle of elevation to the unknown coordinate from each of your known points.
3. Using the angle, evaluate the slope of each line to the unknown point and write an system of equations for this situation.
4. Solve the system of equations for the unknown coordinate. [solve the system of equations by first finding the pseudo-inverse function]



Where you at (alternative activities) ?

Teacher Instructions

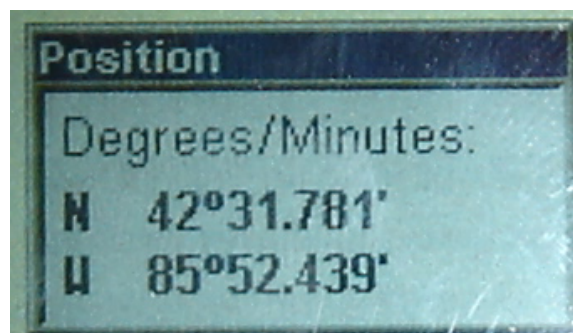
Goal: Apply paper lab activity to finding an unknown position in the field.

Needed Materials

- Student lab handout. (pg 5 and 6)
- **Compass:** Note that bearing can be measure in 1° increments by measuring between markings. To measure bearing, point arrow at target and line up north with the red arrow.



- **GPS** with the ability to get at least 3 decimals into the minute reading.(at least one per class)
Note: New cell-phones come equipped with GPS locaters check with school approval before you use student/teacher cell phones during the school day.



- **Graphing calculator** to assist solving systems of two equations. (optional)

Prior Knowledge:

It is expected that students have done the “Where you at?” paper activity prior to going outside with this activity and have all the prior knowledge associated with that activity.

New information

Compasses measure in degrees clockwise from North (called a bearing). As students do this activity, they will need to convert a bearing measurement into a degree from the positive x-axis, or degrees from East, to allow for easier calculations to be done. It is okay to use negative values for angles from East as when you take the tangent of that angle, it will calculate the appropriate slope.

GPS You will need to work with the given GPS locaters that you have access to. Ideally, you should have the settings to degrees and minutes so that you will be able to get accuracy (In my initial run of this activity there was variation in my school parking lot in the hundredths and tenths place of the minute.)

I allowed students to take measurements as close together as they wanted. It will bring up interesting discussions because the GPS are typically accurate to ≈ 25 -50 feet depending on the make and satellite connections as well as the compass having accuracy to the nearest degree.

The handout for this activity will be mostly for organization. Students should refer to the paper lab activity for how to perform calculations.

Group Members (4 works best, but 5 if necessary)

1 to use GPS

1 to use compass

1 to record

1 to calculate

1 required to oversee accuracy and ask for additional help

A look at error

After you go through the questions on the worksheet. It was informative to use Google Maps by entering in the students coordinates to see where they ended up.

Time requirements: 2-3 days (depending on how many unknown locations you want them to find. 3 points will take a day and a half for data collection and calculations if groups need to share GPS locators.)

Where you at? Group members: _____

Remember to use the GPS to get a coordinate at each spot that you stand. Also make sure you record the bearing at each spot that you stand.

Location #1: _____

x-coordinate	y-coordinate	Bearing	Degrees from East

Equation through 1 st coordinate (explain how you arrived at the equation)	Equation through 2 nd coordinate (explain how you arrived at the equation)
Intersection: (show set-up)	

Location #2: _____

x-coordinate	y-coordinate	Bearing	Degrees from East

Equation through 1 st coordinate	Equation through 2 nd coordinate
Intersection: (show set-up)	

Location #3: _____

x-coordinate	y-coordinate	Bearing	Degrees from East

Equation through 1 st coordinate (show work)	Equation through 2 nd coordinate (show work)
Intersection: (show set-up)	

Discussion questions. Please answer these questions in your group. Each person in your group should be able to answer when called on.

1. Why must you use degrees from East in your calculations of the slope rather than the bearing?
2. Explain what the intersection means for each of the above situations?
3. Is there any error in the intersection you calculated as compared with the “ideal location”? If so, where does it come from and what could be some things that could be done to correct the error? If not, explain how your mad skillz are so perfect!
4. Using this activity to support your reasoning, explain how GPS locators work. What could be done to get a higher degree of accuracy?

Michigan Curriculum Strands (MICLIMB)

Mathematics/Strand I/Content Standard 1/High School/Benchmark 5

Mathematics/Strand I/Content Standard 2/High School/Benchmark 2

Mathematics/Strand I/Content Standard 2/High School/Benchmark 5

Mathematics/Strand I/Content Standard 2/High School/Benchmark 6

Mathematics/Strand II/Content Standard 1/High School/Benchmark 7

Mathematics/Strand II/Content Standard 3/High School/Benchmark 1

Mathematics/Strand II/Content Standard 3/High School/Benchmark 3

Mathematics/Strand II/Content Standard 3/High School/Benchmark 5

Mathematics/Strand II/Content Standard 3/High School/Benchmark 6

Science/Strand I/Content Standard 1/High School

The basic underlying themes of these benchmarks and strands are proper tools of measurements, making direct measurements and calculating indirect measurements, and knowing the limitations of the tools being used.

Tentative Budget

Item	Description	Unit Cost	#	Cost
Protractor		\$0.50	8	\$4.00
String				\$0.00
Washers				\$0.00
Straws				\$0.00
Magnetic Compass		\$10	8	\$80.00
GPS locator	Garmin eTrex	\$92.99	4	\$371.96
Tape Measures	200 ft open real	\$17	2	\$34.00
TOTAL COST				\$489.96