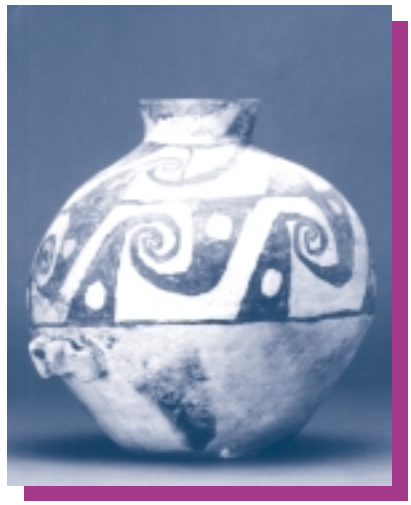
INVESTIGATION 2 Analyzing Matrices

Matrices can be used to organize all sorts of data, not just sales data. In this investigation, you will explore three different situations in which matrices are used to help make sense of data.

Archeologists study ancient people and their cultures. One way they study these people is by exploring sites where they have lived and analyzing objects which they have made. Archeologists use matrices to classify and then compare the objects they find at various archeological sites. For example, suppose that pieces of pottery are found at five different sites. The pottery has certain characteristics: it is either glazed or not glazed, ornamented or not, colored or natural, thin or thick.



1. Information about the characteristics of the pottery at all five sites is organized in the matrix below. A "1" means the pottery has the characteristic and a "0" means it does not have the characteristic.

Pottery Characteristics

	Glaze	Orn	Color	Thin
Site A	0	1	0	0
Site B	1	0	0	0
Site C	1	0	1	0
Site D	1	1	1	1
Site E	0	1	1	1

a. What does it mean for pottery to be "glazed"? "Ornamented"?

b. What does the "1" in the third row and the first column mean?

c. Is the pottery at site E thick or thin?

d. Which site has pottery that is glazed and thick, but is not ornamented or colored?

e. How many of the sites had glazed pottery? Explain how you used the rows or columns of the matrix to answer the question.

2. You can use the matrix to determine how much the pottery differs between sites. For example, the pottery found at sites A and B differ on exactly two characteristics—glaze and ornamentation. So you can say that the *degree of difference* between the pottery at sites A and B is 2.

a. Explain why the degree of difference between pottery at sites A and C is 3.

b. Find the degree of difference between sites D and E.

3. You can build a new matrix that summarizes all the degree of difference information.

a. What number would best describe the difference between site A and site A?

b. What number should be placed in the third row, fourth column? What does this number tell you about the pottery at these two sites?

c. Complete the degree of difference matrix shown below.

Degree of Difference

	A	B	C	D	E
A	___	2	3	___	___
B	2	___	___	___	___
C	3	___	___	___	___
D	___	___	___	___	___
E	___	___	___	___	___

d. Describe at least one pattern you see in the degree of difference matrix.

4. Archeologists want to learn about the civilizations that existed at the sites. For instance, they would like to know whether different sites represent different civilizations and whether one civilization was more advanced than another. A lot of evidence is needed to make such decisions. However, make some conjectures based just on the pottery data in the matrices from Activities 1 and 3.

a. Find two sites that you think might be from the same civilization. Explain how the pottery evidence supports your choice.

b. Find two sites that you think might be from different civilizations. Give an argument defending your choice.

c. Give an argument supporting the claim that the civilization at site D was more advanced than the others. What assumptions are you making about what it means for a civilization to be "advanced"?

5. Matrices also are used frequently by sociologists in their study of social relations. For example, a sociologist may be studying friendship and trust among five classmates at a certain high school. The classmates are asked to indicate with whom they would like to go to a movie and to whom they would loan $5. Their responses are summarized in the following two matrices. ("1" means "yes" and "0" means "no".) Each matrix is read from row to column. For example, the "1" in the first row and fourth column of the movie matrix means that student A would like to go to a movie with student D.

Movie Matrix

Would Like to Go to a Movie (rows) / **with** (columns)

	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	1	0	0	1	0
D	0	1	1	0	1
E	1	0	0	1	0

Loan Matrix

Would Loan Money (rows) / **to** (columns)

	A	B	C	D	E
A	0	0	0	1	1
B	1	0	0	1	0
C	0	0	0	1	0
D	1	1	0	0	1
E	0	1	1	0	0

a. Would student A like to go to a movie with student B? Would student B like to go to a movie with student A?

b. With whom would student D like to go to a movie?

c. What does the "0" in the fourth row, third column of the loan matrix mean?

d. To whom would student A loan $5?

e. A **square matrix** has the same number of rows and columns. The **main diagonal** of a square matrix is the diagonal line of entries running from the top left corner to the bottom right corner. Why do you think there are zeroes for each entry in the main diagonals of the matrices above?

6. Now consider further information conveyed by the matrices above.

a. Explain why the movie matrix could be used to describe *friendship*, while the loan matrix could describe *trust*.

b. Write two interesting statements about friendship and trust among these five students, based on the information in the matrices.

7. Discuss with your group how you can use the rows or columns of the movie and loan matrices to answer the following questions.

a. How many students does student C name as friends?

b. How many students name student C as a friend?

c. Who seems to be the most trustworthy student?

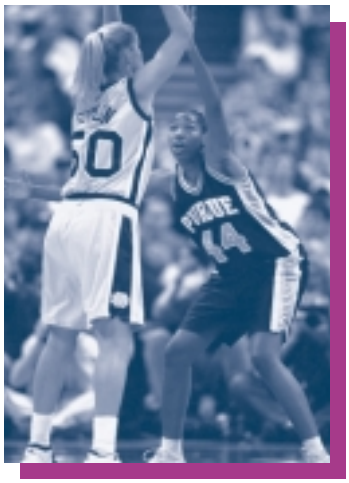
d. Who seems to be the most popular student?

Checkpoint

In the previous investigations, you performed computations on the row or column entries of a matrix to get useful information about the situation being modeled. Give three examples from your analysis of pottery, shoe sales, or friendship and trust showing how you operated on the entries of the given matrix to get additional information. For each example, describe the situation, the computation, and the information obtained.

Be prepared to share your group's examples with the class.

On Your Own



In 2001, the University of Notre Dame won the NCAA women's basketball championship. They completed a 34-2 season by defeating Purdue University in the championship game.

a. Senior Ruth Riley was the high scorer in the championship game for Notre Dame, with 28 points. Teammate Erika Haney scored 13 points. What other factors besides points scored should be taken into account when deciding which player contributed most to the victory?

The matrix below shows some of the non-shooting performance statistics for the seven Notre Dame players who played in the championship game.

Non-Shooting Performance Statistics

	Assists	Steals	Rebounds	Blocked Shots	Turn-overs	Fouls
Haney	2	1	5	1	0	3
Siemon	6	0	9	0	7	3
Riley	1	0	13	7	3	3
Ratay	2	1	4	0	1	4
Ivey	4	6	5	1	4	0
Joyce	1	0	0	0	0	1
Barksdale	0	0	2	2	0	0

Source: und.fansonly.com/sports/w-baskbl/stats/040101aaa.html

b. A "turnover" is when an action (other than fouling or scoring a basket) gives the other team control of the ball. How many turnovers did Ratay have?

c. What is a "rebound"? How many rebounds were made by the Notre Dame players during the game?

d. Which of the performance factors do you think are positive, that is, they contribute toward winning the game? Which performance factors do you think are negative? For the factors that are negative, change the entries in the matrix to negative numbers.

e. Which player had the largest number of positive performance actions?

f. Describe how you could give a "non-shooting performance score" to each player. The score should include both positive and negative factors. Which player do you think contributed most to the game in the area of non-shooting performance? Explain your choice.

INVESTIGATION 3 Combining Matrices

You have seen that a matrix can be used to store and organize data. You also have seen that you can operate on the numbers in the rows or columns of a matrix to get additional information and draw conclusions about the data. Sometimes it is useful to combine matrices, as you will see in the next two situations.

1. Shown below are the movie and loan matrices you analyzed in the previous investigation. Study both matrices to see how friendship and trust are related in this group of five students.

Movie Matrix

Would Like to Go to a Movie	with A	with B	with C	with D	with E
A	0	1	1	1	1
B	0	0	1	1	1
C	1	0	0	1	0
D	0	1	1	0	1
E	1	0	0	1	0

Loan Matrix

Would Loan Money	to A	to B	to C	to D	to E
A	0	0	0	1	1
B	1	0	0	1	0
C	0	0	0	1	0
D	1	1	0	0	1
E	0	1	1	0	0

a. Who does student A consider friends and yet does not trust enough to loan $5?

b. Do you think it is reasonable that a student could have a friend who he or she does not trust enough to loan $5?

c. Who does student B trust and yet does not consider them to be friends? Do you know someone who you trust but who is not a friend?

d. Who does student D trust and also consider to be friends?

2. A friend you trust is a *trustworthy friend*.

a. Combine the movie and loan matrices to construct a new matrix that shows who each of the five students considers to be a trustworthy friend.

b. Write down a systematic procedure explaining how to construct the trustworthy-friend matrix.

c. Compare your procedure with that of other groups.

d. Write two interesting observations about the information in this new matrix.

In the next situation, you will explore other ways of combining matrices and learn some of the ways in which matrix operations are used in business and industry.

In accordance with demand from American consumers, the Big Three automakers in the U.S.—General Motors, Ford, and Chrysler—have been producing many small cars. The matrices below show the production data for 1998 and 1999. Data are given in hundred thousands of small cars produced each quarter (a quarter is three months).

1998 Small Auto Production

	GM	F	C
1st Q	1.53	0.58	0.58
2nd Q	1.39	0.67	0.69
3rd Q	1.30	0.55	0.45
4th Q	1.57	0.46	0.31

1999 Small Auto Production

	GM	F	C
1st Q	1.64	0.54	0.44
2nd Q	1.70	0.73	0.68
3rd Q	1.58	0.41	0.58
4th Q	1.87	0.75	0.63

Source: *Automotive News Market Data Book.* 2000. Crain Communications, Inc.

3. Analyze the two matrices above.

a. How many cars are represented by the first entry in the matrix for 1998?

b. According to these data, how many small cars were produced by Chrysler in the second quarter of 1999?

c. Explain the meaning of the entry in the third row and second column of the 1998 matrix.

4. Additional information can be derived by combining the two matrices.
   a. According to these data, by how much did first-quarter, small-car production increase for GM from 1998 to 1999?
   b. Construct a new matrix with the same row and column labels that shows how much small-car production increased from 1998 to 1999 for each quarter and each manufacturer. Explain any trends or unusual entries.
5. Suppose that the auto industry projected a 10% increase in small-car production from 1999 to 2000, over all quarters and all manufacturers. Construct a matrix that shows the projected 2000 production figures for each quarter and each manufacturer, based on the data given.
6. Construct a matrix with the same row and column labels as the given matrices that shows the total number of small cars produced over the two-year period 1998–99.

Checkpoint

In this investigation, you explored how combining two matrices or multiplying each entry of a matrix by a number helped to derive new information.

a. Which of the activities about small-car production involved combining matrices by adding *corresponding entries*?

b. Which of the activities about small-car production involved combining matrices by subtracting corresponding entries?

c. Which of the activities about small-car production involved multiplying each entry of a matrix by a number?

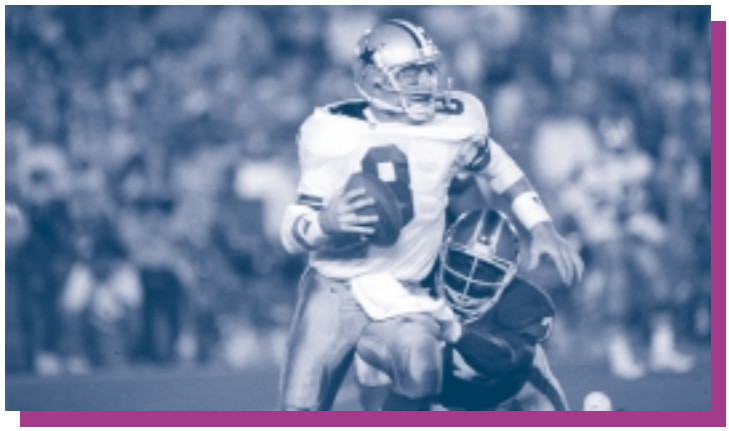
d. Consider all the situations you have analyzed so far. What other operations have you performed on matrices?

Be prepared to explain your group's selections to the entire class.

Several of the operations you have performed on matrices in this investigation are commonly used and have been given standard names. To **add matrices** means to combine them by adding *corresponding entries*. Thus, when adding two matrices you add the entry in the first row and first column from one matrix to the entry in that same position in the other matrix, and then do likewise for the entries in each of the other positions. If A represents one matrix and B represents another matrix, then $A + B$ represents the matrix found by adding the matrices entry by entry. **Subtracting matrices** ($A - B$) is just like adding matrices, except you subtract the corresponding entries instead of adding them. **Multiplying a matrix, *B*, by a number, *k*,** means to multiply *each entry* in the matrix by that number and is represented by kB.

On Your Own

Below are two matrices showing the 1999 and 1998 passing statistics for three top NFL quarterbacks. ("Att" is an abbreviation for "passes attempted"; "Comp" refers to passes completed; "TD" refers to passes thrown for a touchdown; and "Int" refers to passes that were intercepted.)



1999 Passing Statistics

	Att	Comp	TD	Int
Gannon	515	304	24	14
McNair	331	187	12	8
Aikman	442	263	17	12

1998 Passing Statistics

	Att	Comp	TD	Int
Gannon	354	206	10	6
McNair	492	289	15	10
Aikman	315	187	12	5

Sources: *The World Almanac and Book of Facts 2001*. Mahwah, NJ: World Almanac, 2001; *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac, 1999.

Let A represent the 1999 matrix and B represent the 1998 matrix.

a. Compute $A + B$. What does $A + B$ tell you about the passing performance of the three quarterbacks?

b. Compute $A - B$. What does $A - B$ tell you about the passing performance of the three quarterbacks?

c. Compute $B - A$.

- How do the numbers in $B - A$ differ from the numbers in $A - B$?
- What does a negative number in the TDs column of $A - B$ tell you about the trend in touchdown passes from 1998 to 1999?
- What does a negative number in the TDs column of $B - A$ tell you about the trend in touchdown passes from 1998 to 1999?

d. Compute $\frac{1}{2}A$. What could $\frac{1}{2}A$ mean for this situation?