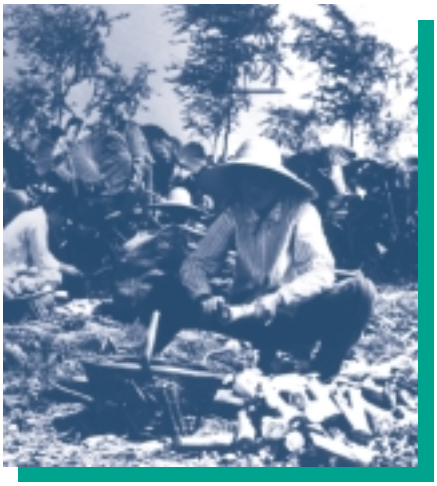
Lesson 1

Simulating Chance Situations

In some cultures, it is customary for a bride to live with her husband's family. Therefore, couples who have no sons and whose daughters all marry will have no one to care for them in their old age.

Customs of a culture and the size of its population often lead to issues that are hard to resolve. China had a population of over 1,200,000,000 in 2000. In an effort to reduce the growth of its population, the government of China instituted a policy to limit families to one child. The policy has been very unpopular among rural Chinese who depend on sons to carry on the family farming.



Think About This Situation

The situation described above raises many interesting mathematical questions as well as societal ones.

a In a country where parents are allowed to have only one child, what is the probability that their one child will be a son? What is the probability they will not have a son? What assumption(s) are you making when you answer these questions?

b If each pair of Chinese parents really had only one child, do you think the population would increase, decrease, or stay the same? Explain your reasoning.

c Describe several alternative plans that the government of China might use to control population growth. For each plan, discuss how you might find the answers to the following questions.

- What is the probability that parents will have a son?
- What will happen to the total population of China?
- What will rural couples think about your plan?

INVESTIGATION 1 How Many Children?

In Part c of the "Think About This Situation" on the previous page, you shared different ways to examine the effects of your policy. In real life, it is hard to gather data that easily show the effects of a policy on the population. It may take several generations to see the long term effects. To estimate these effects you can *simulate* the situation in a way that allows informative data to be gathered more easily and quickly. In this investigation, you will simulate situations by flipping a coin.

1. Suppose China implements a new policy that allows each family to have two children.

a. Explain how to use a coin to simulate the birth of *one* child. What did a head represent? What did a tail represent? What assumption(s) are you making?

b. Explain how to use a coin to simulate the births of *two* children to a family. What are the possible outcomes?

c. When you simulate a family with two children by flipping a coin twice and recording the results, you have conducted one **trial**. To be sure you have a reasonable estimate of what two-child families look like, it is necessary to conduct many trials. Conduct 200 trials simulating two-child families. Share the work among the groups in your class. Make a frequency table like the one below to record the results of your 200 trials.

Type of Family	Frequency
Two Girls	
Older Girl and Younger Boy	
Older Boy and Younger Girl	
Two Boys	
Total Number of Trials	200

d. Use your frequency table to estimate the probability that a family with two children will have *at least one* boy.

e. Estimate the probability that a family with two children will have at least one boy using a mathematical method other than simulation. Explain your other method.

f. Do the four types of families—two girls, older girl and younger boy, older boy and younger girl, two boys—appear to be **equally likely**? Describe the meaning of *equally likely* for a friend who is not in this class.

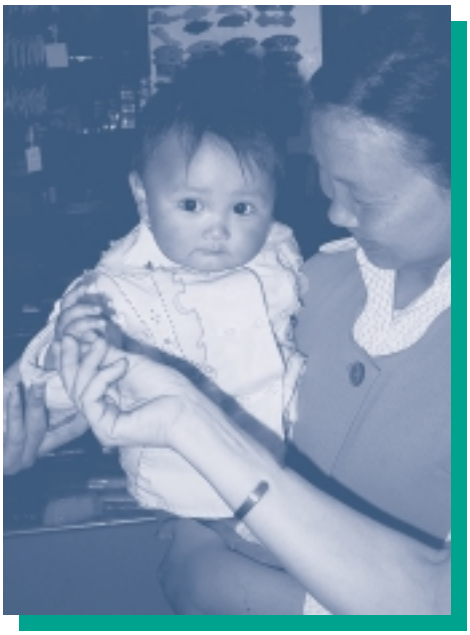
g. What is the total number of children in the 200 trials in Part c? What is the total number of girls? Of boys?

Here is one plan for reducing population growth that your class may have discussed.

Allow parents to continue to have children until a boy is born.
Then no more children are allowed.

For most of the remainder of this investigation, you will examine this plan. You will begin by making your best prediction about the effects of such a policy. Then you will use simulation techniques to improve your estimates.

2. Suppose that in rural China all parents continue having children until they get a boy. After the first boy, they have no other children. In your group, discuss each question below. Write your best prediction of the answer to each question.



a. Will more boys or more girls be born in rural China?

b. What will be the average number of children per family in rural China?

c. Will the population of rural China increase, decrease, or stay the same?

d. What percentage of families will have only one child?

e. What percentage of families will have four children or more?

f. What percentage of the children in rural China will belong to single-child families?

To get a good estimate of the answers to the questions in Activity 2, your group could simulate the situation. To do this, design a **simulation model** that imitates the process of parents having children until they get a boy.

3. Explain how to use a coin to conduct one trial that models a family having children until they get a boy.

a. Carry out one trial for your simulation of having children until a boy is born. Make a table like the following one. Make a tally mark (/) in the frequency column opposite the number of tosses it took to get a "boy."

Number of Tosses to Get a "Boy"	Frequency
1	
2	
3	
4	
5	
6	

Number of Tosses to Get a "Boy"	Frequency
7	
8	
9	
10	
⋮	
Total Number of Trials	50

b. Continue carrying out trials of having children until a boy is born. Stop when you have a total of 50 "families." Divide the work among the members of your group. Record your results in the frequency table. Add as many additional rows to the table as you need.

- How many of your 50 families had four children or more?
- How many boys were born in your 50 families?
- How many girls were born in your 50 families?

4. Now use your frequency table to estimate answers to the six questions posed in Activity 2.

a. Compare your estimates with your original predictions. For which questions did your initial prediction vary the most from the simulation estimate? (If most of your original predictions were not accurate, you are in good company. Most people aren't very good at predicting the answers to probability problems.)

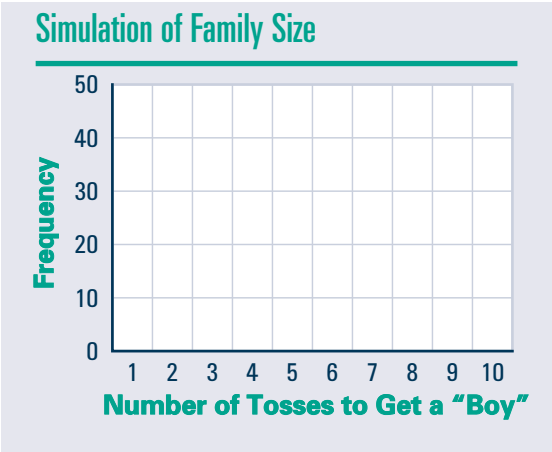
b. Write several misconceptions that you or others in your group originally had about this situation.

5. Make a histogram of your group's frequency table on a graph like the one shown below or on your calculator or computer.

a. Describe the shape of this distribution.

b. What is the largest family size? The smallest?

c. On the histogram, locate the median and the lower and upper quartiles of the distribution.



6. Each group should reproduce their histogram on a sheet of paper or on the chalkboard.

a. Describe any similarities in the histograms.

b. Describe any differences in the histograms. Explain why the differences occurred.

c. Combine the frequency tables from all of the groups in your class into one frequency table on the overhead projector or chalkboard.

d. Make a histogram of the combined frequency table. How are the histograms from the individual groups similar to this one? How are they different?

e. Reproduced below are the questions from Activity 2. Estimate the answers to these questions using the combined frequency table of Part c above.

- Will more boys or more girls be born in rural China?
- What will be the average number of children per family in rural China?
- Will the population of rural China increase, decrease, or stay the same?
- What percentage of families will have only one child?
- What percentage of families will have four children or more?
- What percentage of the children in rural China will belong to single-child families?

f. Should you have more confidence in the estimates from Part e or in the estimates from your group? Explain your reasoning.

7. In the "Think About This Situation" on page 484, your class proposed several alternative plans for reducing population growth in China.

a. As a class, choose a plan different from the one in which parents have children until they get a boy, and design one trial of your plan.

b. Perform at least 200 trials, sharing the work among groups in the class. Place your results in a frequency table.

c. Under your plan, what is the probability that parents will have a son? How did you calculate the probability?

d. Will the population of China increase, decrease, or remain the same under your plan? Explain your reasoning.

Checkpoint

In this lesson, you learned how to design simulations.

a Describe, in your own words, what a *simulation model* is. Why is it important to conduct a large number of trials?

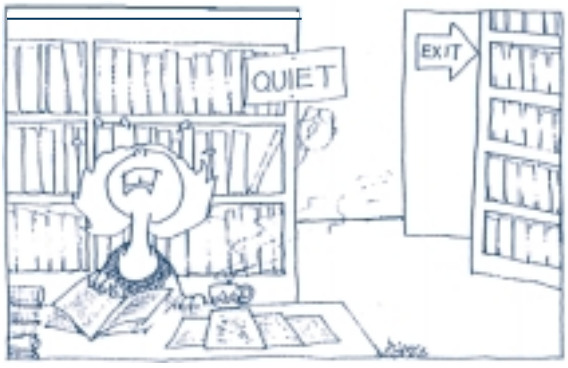
b Why is it always a good idea to make a histogram of the results of a simulation?

c Describe a way to simulate the have-children-until-you-get-a-boy plan that does not use coins.

d If you flip a coin until you get a head and then repeat this many times, will you tend to have a larger proportion of heads or of tails?

Be prepared to share your descriptions and thinking with the class.

Simulation is a good way to estimate the answer to a probability problem. The greater the number of trials, the more likely it is that the estimated probability is close to the actual probability. In our complex world, simulation is often the only feasible way to deal with a problem involving chance. Simulation is an indispensable tool to scientists, business analysts, engineers, and mathematicians.



"I've had it! Simulated wood, simulated leather, simulated coffee, and now simulated probabilities!"

STATISTICS: CONCEPTS AND CONTROVERSIES by Moore © 1979 by W.H. Freeman and Company. Used with permission.

On Your Own

When asked in what way chance affected her life, a ninth-grader in a very large Los Angeles coeducational city high school noted that students are chosen randomly to be checked for weapons. Suppose that when this policy was announced, a reporter for the school newspaper suspected that the students would not be chosen randomly, but that boys would be more likely to be chosen than girls. The reporter then observed the first search and found that all 10 students searched were male.

a. If a student is in fact chosen randomly, what is the probability that the student will be a boy?

b. Write instructions for conducting one trial of a simulation that models selecting 10 students at random and observing whether each is a boy or a girl.

c. What assumptions did you make in your model?

d. Perform 20 trials using your simulation model.

e. Report the results in a frequency table showing the number of boys selected.

f. Write an article for a school paper describing your simulation, its results, and your conclusion. Include a histogram in your article.