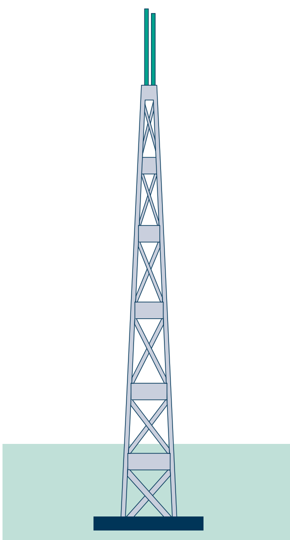
Lesson 2

Managing Conflicts

Have you ever noticed how many different radio channels there are? Each radio station has its own transmitter which broadcasts on a particular channel, or frequency.

The Federal Communications Commission (FCC) makes sure that the broadcast from one radio station does not interfere with the broadcast from any other radio station. This is done by assigning an appropriate frequency to each station. The FCC requires that stations within transmitting range of each other must use different frequencies. Otherwise, you might tune into "ROCK 101.7" and get Mozart instead!



Think About This Situation

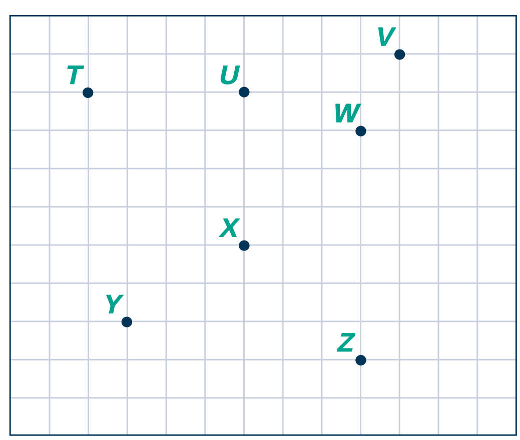
Seven new radio stations are planning to start broadcasting in the same region of the country. The FCC wants to assign a frequency to each station so that no two stations interfere with each other. The FCC also wants to assign the fewest possible number of new frequencies.

What factors need to be considered before the frequencies can be assigned?

b What method can the FCC use to assign the frequencies?

INVESTIGATION 1 Building a Model

Suppose that because of geographic conditions and the strength of each station's transmitter, the FCC determines that stations within 500 miles of each other must be assigned different frequencies. Otherwise their broadcasts will interfere with each other. The locations of the seven stations are shown on the grid at the right. A side of each small square on the grid represents 100 miles.



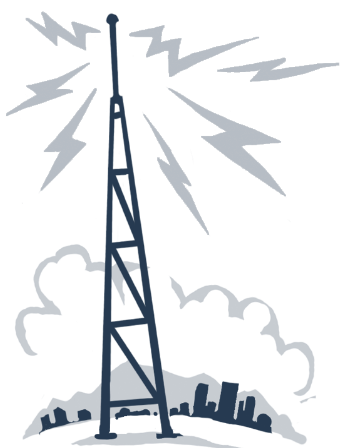
Scale: ⊢⊣ **= 100 miles**

1. Working on your own, figure out how many different frequencies are needed for the seven radio stations. Remember that stations 500 miles or *less* apart must have different frequencies. Stations more than 500 miles apart can use the same frequency. *Try to use as few frequencies as possible.*

2. Compare your answer with others in your group.

a. Did everyone use the same number of frequencies? Reach agreement in your group about the minimum number of frequencies needed for the seven radio stations.

b. Suppose one person assigns two stations the same frequency and another person assigns them different frequencies. Is it possible that both assignments are acceptable? Explain.



In this case, it is possible to find the minimum number of frequencies by trial and error. What would you do when there are many more radio stations? A more systematic method is needed for more complicated situations. You could begin by modeling the problem with a graph similar to the graphs in the previous lesson. Remember, *to model a problem with a graph, you must first decide what the vertices and edges represent.*

3. Working on your own, begin modeling this problem with a graph.

a. What should the vertices represent?

b. How will you decide whether or not to connect two vertices with an edge? Complete this statement:

Two vertices are connected by an edge if . . . .

c. Now that you have specified the vertices and edges, draw a graph for this problem.

4. Compare your graph with others in your group.

a. Did everyone in your group define the vertices and edges in the same way? Discuss any differences.

b. For a given situation, suppose two people define the vertices and edges in two different ways. Is it possible that both ways accurately represent the situation? Explain your reasoning.

c. For a given situation, suppose two people define the vertices and edges in the same way. Is it possible that their graphs look different but both are correct? Explain your reasoning.

5. A common choice for the vertices is to let them represent the radio stations. Edges might be thought of in two ways as described in Parts a and b below.

a. You might connect two vertices by an edge whenever the stations they represent are 500 miles or *less* apart. Did anyone in your group do this? If not, draw a graph where two vertices are connected by an edge whenever the stations they represent are 500 miles or *less* apart.

b. You might connect two vertices by an edge whenever the stations they represent are *more* than 500 miles apart. Did anyone in your group do this? If not, draw a graph where two vertices are connected by an edge whenever the stations they represent are *more* than 500 miles apart.

c. Compare the graphs from Parts a and b.

- Are both graphs accurate ways of representing the situation?
- Which graph do you think will be more useful and easier to use as a mathematical model for this situation? Why?

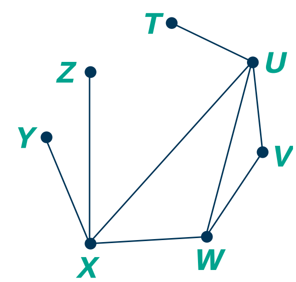
6. For the rest of this investigation, you will use the graph where edges connect vertices that are 500 miles or less apart. Make sure you have a neat copy of this graph.

a. Are vertices (stations) *X* and *W* connected by an edge? Are they 500 miles or less apart? Will their broadcasts interfere with each other?

b. Are vertices (stations) *Y* and *Z* connected by an edge? Will their broadcasts interfere with each other?

c. Compare your graph to the graph at the right.

- Does this graph also represent the radio-station problem?
- What criteria can you use to decide if two graphs both represent the same situation?

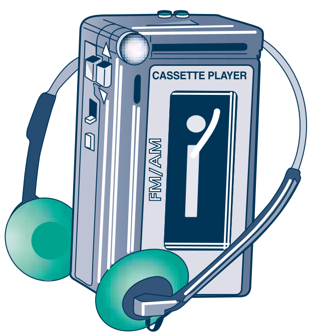


7. So far you have a model that shows all the radio stations and which stations are within 500 miles of each other. The goal is to assign frequencies so that there will be no interference between stations. You still need to build the frequencies into the model. So, as the last step in building the graph model, represent the frequencies as **colors**. To **color a graph** means to assign colors to the vertices so that two vertices connected by an edge have different colors.

You can now think about the problem in terms of *coloring the vertices of a graph.* The following table contains statements about stations and frequencies in the left-hand column. Corresponding statements about vertices and colors are in the right-hand column. Write statements to complete the right-hand column of the table.

Statements about stations and frequencies	Statements about vertices and colors
Two stations have different frequencies.	Two vertices have different colors.
Find a way to assign frequencies so that stations within 500 miles of each other get different frequencies.	
Use the fewest number of frequencies.	

8. Now use as few colors as possible to color your graph for the radio station problem. That is, assign a color to each vertex so that any two vertices that are connected by an edge have different colors. You can use colored pencils or just the names of some colors to do the coloring. Color or write a color name next to each vertex. Try to use the smallest number of colors possible.



9. Compare your coloring with that of another group.

a. Do both colorings satisfy the condition that vertices connected by an edge must have different colors?

b. Do both colorings use the same number of colors to color the vertices of the graph?

c. Reach agreement about the minimum number of colors needed. Explain, in writing, why the graph cannot be colored with fewer colors.

d. Suppose one group assigns two vertices the same color and another group assigns them different colors. Is it possible that both assignments are acceptable? Why or why not?

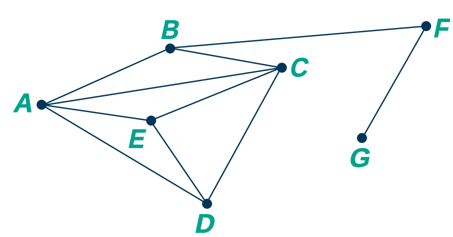
e. What is the connection between graph coloring and assigning frequencies to radio stations? As you answer this question, compare the results of this activity to those in Activity 2.

10. Think about the strategy you used in Activity 8 to color the radio station graph with as few colors as possible.

a. Write down a step-by-step description of your coloring strategy. Write the description so that your strategy can be applied to graphs other than just the radio station graph.

b. Use the description of your strategy to color a copy of the graph at the right.

c. Refine the directions for your coloring strategy so that any one of your classmates could follow the directions.



11. Exchange your written coloring directions with another group. Then do the following:

a. Use the other group's directions to color a second copy of the graph in Part b of Activity 10. The other group will be doing the same thing with your directions.

b. Compare your colorings with the other group's colorings.

- Are they the same?
- Are they each legitimate colorings?
- Do they each use the least number of colors possible? Reach agreement with the other group about the minimum number of colors needed to color the graph.

c. Discuss any problems that came up with either group's coloring directions. If necessary, rewrite your directions so that they work better and are easier to follow.

As you saw in the previous lesson, a careful list of directions for carrying out a procedure is called an *algorithm*. Designing and applying algorithms is an important method for solving problems. There are many possible algorithms for coloring the vertices of any graph, including the ones you developed.

Checkpoint

Some problems can be solved by coloring the vertices of an appropriate graph model.

a What do the vertices, edges, and colors represent in the graph model that you have been using for the radio station problem?

b How does "coloring a graph" help solve the radio station problem?

c In what ways can two graph models differ and yet still both accurately represent a given situation?

d What are some strengths and weaknesses of the graph-coloring algorithm created by your group?

Be prepared to share your thinking and coloring algorithm with the entire class.

Graph-coloring algorithms continue to be an active area of mathematical research with many applications. It has proved quite difficult to find an algorithm that colors the vertices of any graph using as few colors as possible. You often can figure out how to do this for a given small graph, as you have done in this investigation. However, no one knows an efficient algorithm that will color *any* graph with the *fewest* number of colors. This is a famous unsolved problem in mathematics. At the time this book was written, at least, the problem was still unsolved. . . .