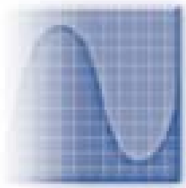
CORE-PLUS MATHEMATICS PROJECT

A Balanced Approach to Mathematics Education

Contemporary Mathematics in Context

A Unified Approach



A Balanced Approach to Mathematics Education

Facts about Contemporary Mathematics In Context *from the Core-Plus Mathematics Project*

Effective mathematics education requires a balanced approach to both content and teaching. Educational programs have too often over-emphasized one component of mathematics education at the expense of another. The *Contemporary Mathematics in Context* (*CMIC*) curriculum provides a balanced approach centered around fundamental mathematical content that is organized and presented in a way that promotes effective teaching and learning. Each section of this brochure identifies one important aspect of contemporary mathematics education, and describes how it is addressed by *CMIC*.

Integrated Mathematics

An integrated mathematics curriculum is an effective way to provide successful mathematics education.

Contemporary Mathematics in Context *is an integrated mathematics curriculum.*

- Most countries that outperform the US in pre-college mathematics education use an integrated curriculum (US TIMSS National Research Center, 1998).
- A report analyzing the below-average performance of US mathematics students on the Third International Mathematics and Science Study (TIMSS), states:

  "How mathematics is arranged in courses [in the US] also seems to be part of the problem. . . . US mathematics students take separate courses in geometry, pre-calculus, etc. In most TIMSS countries, students take a course in mathematics—a course which may include parts of advanced algebra, geometry, finite mathematics, and calculus at the same time." (US TIMSS National Research Center, 1998, p. 5)

  Each year of the *CMIC* curriculum consists of four interwoven "strands" of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Algebra and geometry are the most dominant strands, but significant amounts of statistics, probability, and discrete mathematics are also included since these areas of mathematics are increasingly important in the high-tech information age in which we live. The conceptual underpinnings of calculus, such as rate of change and accumulated change, are also developed.
- Problem solving, conceptual understanding, and basic skills are all developed in the context of real-world applications that naturally cut across the different strands of mathematics.

Algebra

Algebra is the core of high school mathematics. A good high school mathematics program must provide solid, comprehensive instruction in algebra.

Algebra is developed systematically and rigorously in the* CMIC *curriculum. The* CMIC *curriculum teaches more algebra to more students.

- There are three fundamental ways to represent algebraic relationships: tables, graphs, and equations. *CMIC* students develop the ability to model and solve problems using all three representations.
- In Course 1, there is more emphasis on using tables and graphs to solve problems and to develop conceptual understanding, with less emphasis on analysis of equations and other algebraic expressions. Based on the solid conceptual understanding that is developed in Course 1 and continued through later courses, students do more extensive and rigorous algebraic work in Courses 2, 3 and 4. Special attention is paid in Course 4 to completing the development of all algebraic skills that are necessary as preparation for Calculus and to practicing the algebraic skills needed for college placement exams.

Basic Skills

Basic skills are an essential part of a good mathematics program.

More basic skills are taught in the* CMIC *curriculum than has been common in the past.

- The *Contemporary Mathematics in Context* curriculum not only teaches basic skills in algebra and geometry, but also important skills in statistics, probability, and discrete mathematics.
- Instructional time is very valuable, and there is never enough of it. The *CMIC* author team, in consultation with teachers and an international advisory board, made careful decisions about what to include, what to emphasize, and what to delete.
- *Contemporary Mathematics in Context* teaches basic skills all the time and at the right time. For example, rather than do a heavy treatment of algebraic skills early (like simplifying and factoring expressions and solving equations), it is more effective to develop the skills gradually as students mature mathematically.
- The *CMIC* curriculum takes a balanced approach to basic skills. An over-emphasis on procedural skills has been tried and it does not work. This was seen on a national scale in the 1970s, during the so-called "back to basics" movement. After a decade of heavy emphasis on procedural algebraic skills, national tests showed that student achievement related to these skills declined or stayed the same (Kenney and Silver, 1997), and performance on tests of problem solving was very poor (Carpenter et al., 1981). More recently, results of the Third International Mathematics and Science Study show that US texts and teachers have spent a large amount of time on procedural skills, in stark contrast to countries around the world that outperform the US in mathematics education, in which more emphasis is placed on conceptual understanding and problem solving (TIMSS US National Research Center, 1998). As a further indication that the traditional curriculum has not been successful, The Center for Educational Reform, a conservative think tank, reports that more than 20 million Americans since 1983 have reached the 12th grade unable to do basic math (as reported in *USA Today*, 14 July 1998, page 2A). Students using the *CMIC* curriculum learn basic skills based on a foundation of conceptual understanding, and they apply the skills to solve significant problems.

Practice

Students need practice to learn mathematics, and it's best when practice is distributed over time.

Too much emphasis on drill and practice turns mathematics learning into rote memorization of meaningless procedures. Too little emphasis on drill and practice results in lack of learning. The* CMIC *curriculum has a healthy emphasis on drill and practice.

- *CMIC* students are actively engaged in developing robust understanding of mathematical concepts and methods. This is supported by appropriate drill and practice that is built into the lessons. Homework problems provide additional practice. Distributed practice and review of important concepts and skills are provided in *Reference and Practice* books. In addition, targeted practice sheets are provided as part of the Teacher Resource materials, to be used at each teacher's discretion.
- In the *CMIC* curriculum, practice is distributed across the different strands of mathematics and across the years of the curriculum.
- The term "drill and practice" typically refers to skills. In the *CMIC* curriculum there is also drill and practice on concepts.
- *CMIC* students learn through a balanced approach of concept development, practice with important procedures, and problem solving.

Logic and Proof

A good mathematics program must teach reasoning and proof.

The* CMIC *curriculum recognizes that logic and proof are cornerstones of mathematics.* CMIC *students develop the ability to reason logically and carry out formal proofs.

- From the beginning of Course 1, *CMIC* students are expected to explain, justify, defend, and conjecture and verify, both verbally and in writing.
- Proof in geometry, as well as proof in algebra, statistics, trigonometry, and discrete mathematics, is carefully developed in Courses 3 and 4. For example, students write algebraic proofs in the *Symbol Sense and Algebraic Reasoning* unit, geometric proofs in the *Shapes and Geometric Reasoning* unit, proofs of trigonometric identities in the *Functions and Symbolic Reasoning* unit, and proofs by mathematical induction in the unit on *Counting Models*.

Theory and Applications

Good mathematics programs have a balance of applications and theory.

The CMIC curriculum features mathematics in context. Thus, mathematics becomes relevant and sensible to students, and they learn more effectively. However, the mathematics, not the application, is on center stage.

- While developing mathematics in the context of applications, *CMIC* units are not thematic. Each *CMIC* unit is centered on key mathematical ideas, rather than on a particular application. In a *CMIC* unit, students investigate a fundamental mathematical concept, like linear functions or statistical correlation, in the context of a variety of rich real-world applications.
- A common criticism of mathematics education is that "textbook math often remains disconnected from real-life math" (*Wall Street Journal*, 6 June 1998, page A15). This is not the case in *Contemporary Mathematics in Context*, since most of the mathematics is developed in real-world contexts.
- In addition to developing mathematics in context, theory and formal development occur throughout the curriculum in specific units, lessons, and homework sections. For example, matrix algebra is studied in a Course 2 unit. Manipulating symbolic expressions into equivalent forms to reveal important properties of relationships, to solve equations and inequalities, and to prove generalizations is studied in several units in Courses 3 and 4. Problems within the *Organizing* section of every homework section focus on the formal mathematics underlying the mathematical concepts and models developed in each lesson.

Technology

Technology is a powerful aid to education, but it must be used wisely.

The CMIC curriculum uses technology as a tool to promote thinking, never as a crutch to replace thinking. In particular, graphing calculators are used judiciously as tools to promote conceptual understanding, problem-solving ability, and skill development.

- A general principle for the use of calculators or computers in the *CMIC* curriculum is that first students develop conceptual understanding, then they use technology to extend and deepen their understanding and to solve more complex problems. Judging the reasonableness of technology-produced results is emphasized.
- The use of calculators and/or computers permits the curriculum and instruction to emphasize multiple representations (numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking is central.
- The following features of calculators and/or computers are emphasized in the *CMIC* curriculum:
  - Graphics capabilities — to draw and investigate graphs in algebra, trigonometry, geometry, probability, and statistics;
  - Programming capabilities — to create basic computer programs to investigate mathematical situations;
  - Table-building capabilities — to detect numerical patterns and make connections to graphs and equations;
  - Spreadsheet-like capabilities — to explore processes of sequential change like population growth, compound interest, or chemical concentration;
  - Data analysis capabilities — to plot and summarize data, find regression equations, and to simulate complicated probabilistic situations;
  - Numerical capabilities — to explore and solve real-world problems with "messy" numbers.

Effective Teaching Methods

A good mathematics program must promote a research-based and balanced approach to teaching.

The CMIC curriculum is designed to support active learning and teaching, because research shows that this is most effective for successful mathematics learning. Active learning takes place through a variety of teaching methods — collaborative problem solving, teacher-directed instruction, whole-class discussion, and individual practice.

Each multi-day *CMIC* lesson consists of five phases — Launch, Investigate, Checkpoint, On Your Own, and Homework.

- Launch — Each lesson begins with a teacher-directed whole-class discussion of an initial problem situation and related questions.
- Investigate — Classroom activity then shifts to guided student investigations of the target concepts, as motivated and introduced in the launching situation. This is most often done through student collaboration in small groups, but is sometimes teacher-directed.
- Checkpoint — Following each investigation is a Checkpoint. A Checkpoint consists of key summarizing questions. Students answer and analyze these questions in a whole-class discussion moderated by the teacher. During this time in particular, students summarize and consolidate their learning. Summaries are recorded in student Math Toolkits for future reference.
- On Your Own — At the end of each Investigation, students solve a few problems on their own to assess the understanding and skills they have developed.
- Homework — Students work individually on four general types of homework tasks. *Modeling* problems allow students to practice the ideas and methods developed in the lesson in new or related contexts. *Organizing* problems focus on the formal mathematics and connections among strands. *Reflecting* tasks allow students to monitor and evaluate their understanding. *Extending* problems encourage further, deeper, or more formal study of the topics under investigation.

Access and Challenge

A good mathematics curriculum must be both challenging and accessible, thereby providing a solid mathematics education for all students.

The CMIC curriculum makes more mathematics accessible to more students, and at the same time challenges even the most able students.

- In our contemporary high-tech society, more students need more mathematics. The *CMIC* curriculum is designed to provide a solid mathematics education for work-prep, tech-prep, and college-prep students. Differences in student background, performance, and interest are accommodated by the depth to which investigations may be pursued, by the rich variety of homework problems, and by the nature and complexity of applications.
- The *CMIC* curriculum has been used successfully with students ranging from remedial students who have never (yet) been successful in mathematics to top students in math/science magnet schools and in schools offering the International Baccalaureate Program.

Preparation for College

High school mathematics education must provide a solid preparation for college.

The* CMIC *curriculum prepares students to pursue college majors in mathematics and science, as well as in any non-technical field.

- The *CMIC* curriculum provides the rigorous content needed for students to succeed in any freshman-level college mathematics course, including calculus and statistics.
- While still in high school, students are prepared to take Advanced Placement (AP) Statistics after 3 years of the *CMIC* curriculum, and accelerated students who complete Course 4 by the end of their junior year are prepared to take AP Calculus as seniors.
- Evaluation results show that *CMIC* students significantly outperform non-*CMIC* students on the mathematics subtest of the Iowa Tests of Educational Development. This test correlates even more strongly with success in college than the ACT or SAT tests.
- Evaluation results show that *CMIC* students do as well or better than non-*CMIC* students on the SAT and ACT college entrance examinations.
- Evaluation results show that on a mathematics department placement test at a major Midwestern university, *CMIC* students perform as well as students in traditional precalculus courses on algebra and advanced algebra subtests, and they perform significantly better on the calculus readiness subtest.
- *CMIC* students have been accepted at over 500 schools around the country, including Harvard University, Stanford, Duke University, Massachusetts Institute of Technology (MIT), Notre Dame, the University of Michigan, the University of Chicago, the University of California at Berkeley, Clemson University, the University of Virginia, Purdue University, Boston College, the University of Wisconsin-Madison, Rice University, the University of Washington, Georgetown University, the Air Force Academy, Northwestern University, Morehouse College, the University of Arizona, Vanderbilt University, the University of Hawaii, and Pennsylvania State University.

Research Results

A good mathematics program should have rigorous research that documents its effectiveness.

Extensive research shows that the* CMIC *curriculum works. Large-scale, rigorously conducted research studies show that the* CMIC *curriculum effectively promotes student learning.

- *CMIC* students perform well on standardized tests:
  - **Iowa Tests of Educational Development** — *CMIC* students significantly outperform non-*CMIC* students on the mathematics subtest of the Iowa Tests of Educational Development.
  - **SAT and ACT** — *CMIC* students do as well or better than non-*CMIC* students on the SAT and ACT college entrance exams.
  - **National Assessment of Educational Progress** — On a test consisting of released twelfth-grade items from the National Assessment of Educational Progress (NAEP), *CMIC* students' performance at the end of Course 3 was considerably higher than a nationally representative sample of twelfth-grade students in all of the following areas: statistics and probability, measurement, algebra and functions, geometry, numbers and operations, conceptual understanding, problem solving, and procedural skill.
- On a survey developed by the Core-Plus Mathematics Project, *CMIC* students at the end of Course 2 had better attitudes and beliefs about mathematics than non-*CMIC* students. Between 65% and 80% of *CMIC* students agreed or strongly agreed that they understood the mathematical ideas, felt confident in their problem-solving ability, learned to reason mathematically, learned more mathematics by using the calculator, became better at talking and writing about mathematics, felt that group work helped them learn mathematics, and wanted to take a math course taught in the same way next year.
- Research findings have appeared in articles in peer-reviewed journals, in peer-reviewed book chapters, in papers presented at research conferences, in CPMP field-test reports, and in doctoral dissertations. For details of these studies and for evaluation reports from individual school districts, visit www.wmich.edu/cpmp.