Examples of Tasks from Course 2, Unit 7

What Solutions are Available?

Lesson 1: page 466, Modeling Task 1; page 466, Modeling Task 3; page 468, Organizing Task 3; page 469, Extending Task 1

Lesson 2: page 477, Modeling Task 1; page 478, Modeling Task 4; page 479, Organizing Task 3; page 480, Organizing Task 4

Lesson 3: page 491, Organizing Task 4; page 503, Modeling Task 2; page 505, Organizing Task 1

These tasks are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing your child's understanding and independence.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Statistics page or the Scope and Sequence might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 7

Upon completion of this unit, students should be able to:

- construct an approximate probability distribution from the results of a simulation
- use a probability distribution to understand and analyze the situation being modeled
- explore various shapes of distribution, in particular the geometric distribution
- find the mean (expected value) of a probability distribution
- recognize rare events in a probability distribution
- understand what is meant by "independent" events
- calculate the probability of two independent events using the Multiplication Rule

Selected Homework Tasks and Expected Solutions

(These solutions are for problems in the book with 2003 copyright. If a student is using a book with an earlier copyright, you may notice that the problems don't match exactly, although the intent of the problems should be the same.)

Lesson 1, page 466, Modeling Task 1

a.
- 1. You might get a heart on the first draw.
- 40. You might draw 13 diamonds, 13 spades, 13 clubs and then draw a heart.
- No. If you do not replace the cards that are not hearts, then there are fewer and fewer total cards in the pack, and still the 13 hearts. So, the probability of getting a heart increases each time a non-heart is drawn and discarded.

b. Answer left to students.

c. Answer left to students.

Lesson 1, page 466, Modeling Task 3

a. The first thing to do is to make the underlying probability explicit. In this case, there are 4 different stickers, each equally likely to occur, so there is a probability of 0.25 that any randomly selected box will contain a particular sticker. You need a model that displays the same characteristic. You could use a spinner with 4 equal parts, or a random number table, or a random number generator on the calculator. Assign the meaning "bird of paradise" to 1, "tiger" to 2, "elephant" to 3, "crocodile" to 4. For example, 2, 2, 4, 2, 1 would mean that a bird of paradise sticker was obtained in the 5th box. (If you do not have access to a random number generating device, you could roll a die and ignore the 5 and 6 rolls, or place 4 different-colored items in a bag and draw for a sticker.)

Students learned how to set up simulations to make frequency distributions in Course 1 Unit 7.

b. Answer left to students.

c. Answer left to students.

d. To compute the average number of boxes purchased, divide the *total* number of boxes purchased by the number of trials. For example, in the incomplete table in Part b there are 19 trials in which only 1 box was bought, but there are 14 trials in which 2 boxes were bought. From these two rows alone there is a total of 19 + 28 boxes, over the course of 19 + 14 trials. That is, you multiply the number of boxes by frequency to get the total number of boxes. Dividing this number by the sum of the frequency column should result in about 4. A person can expect to have to buy 4 boxes to get a bird of paradise sticker.

e. According to the table (before other trials are added), there are 9 trials in which more than 10 boxes were purchased. This is about 10% of the 95 trials. (Answers will vary because students will add additional trials to the table.)

f. If you consider "5%" as the cutoff for rare events then a "wait" of 10 boxes is not rare. That is, it happens, purely by chance, about 10% of the time.

Lesson 1, page 468, Organizing Task 3

As explained in Modeling Task 3 above, you can use the random number generator on the calculator to mimic events with specific probabilities in the real world. An icosahedral die has 20 faces, and each is equally likely to occur. Therefore, you need to generate the integers 1–20 at random. Different calculators have different ways to do this but a common calculator uses **randInt(1,20)**.

Lesson 1, page 469, Extending Task 1

You would have to be sure that the position of the rear tire is not dependent on the position of the front tire. For example, if the tires start at 3:00 and 10:00, then will they always be 7 "hours" apart? As it turns out, the positions are almost independent. So, this model implies that the two numbers are generated independently of each other. Assign the numbers 1–12 for each of the positions on the front tire, and 1–12 for each of the positions on the rear tire. Then generate a pair of numbers at random. For example, if the numbers generated are 3 and 4 we know that the front tire was at 3:00 and the rear at 4:00. Repeat this trial many times and record how many of the trials matched the 3:00 and 10:00 that the officer recorded. You can do this by using **randInt(1,12)** and recording pairs of numbers, or you can use **randInt(1,12,2)** to produce pairs of values.

Lesson 2, page 477, Modeling Task 1

a. The probability of any particular "hour" for the front tire is $\frac{1}{12}$. Therefore, the probability that the front tire will, by chance, match the original 3:00 is $\frac{1}{12}$. Likewise for a match for the rear tire at 10:00. The probability that *both* will match is $\left(\frac{1}{12}\right)\left(\frac{1}{12}\right) = \frac{1}{144}$.

You could make an area model to illustrate this, instead of applying a rule. Imagine a square with each side representing a tire and subdivided into 12 sections, labeled 1–12. Then there would be 144 small squares inside this, one for each combination. There would be only 1 small square associated with 3 for front and 10 for rear. The area model was employed in class as a way to show when *it is appropriate to multiply probabilities, and* why *multiplying probabilities makes sense. Students should not be operating with memorized rules, without understanding how these rules are developed and when to apply them.*

b. (See Extending Task 1 above.)

c. Answer left to students.

Lesson 2, page 478, Modeling Task 4

a. Answer left to students.

b. An area model (as seen at the right) shows the equally likely possibilities. The "R" represents the dominant tongue-rolling gene and the "r" represents the non-dominant gene. Three fourths of the area will contain an "R" and so three fourths of children (born by parents who each have exactly one tongue-rolling gene) will inherit this trait. The probability for any one child is 75%.

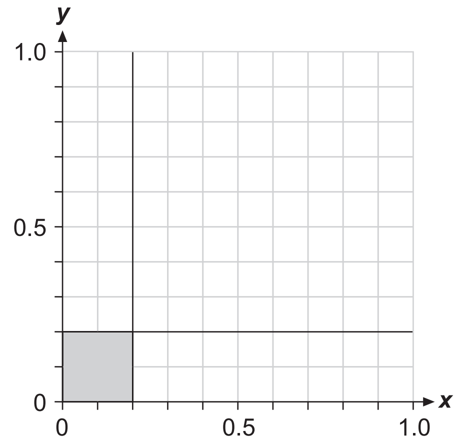
c. Answer left to students.

Tongue Roll Father \ Tongue Roll Mother	R	r
R	RR	Rr
r	rR	rr

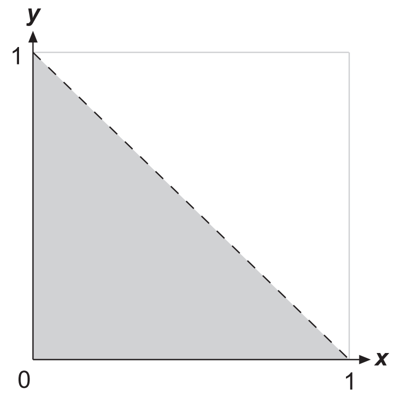
Lesson 2, page 479, Organizing Task 3

You cannot list and count the possible numbers, but this geometric model can be used.

a. The shaded area represents $x < 0.2$ and $y < 0.2$.



b. This time you have the condition that $x + y < 1$. The diagonal line represents possible (x, y) pairs where $x + y = 1$, or $y = 1 - x$. The area under the diagonal line represents the probability that the sum of the two numbers is less than 1. Computing the area of the triangle is left for the student to complete.



c. Answer left to students.

d. Answer left to students.

Lesson 2, page 480, Organizing Task 4

a. The notation "$P(A|B)$" means "probability that A occurs, given that B occurrs." Suppose A is the event "rolls doubles" and B is the event "rolls odd sum." $P(A) = \frac{1}{6}$. $P(B) = \frac{18}{36}$. But if the roll has already resulted in B, then $P(\textit{doubles}|\textit{odd sum}) = 0$. So, $P(A) > P(A|B)$, for these two events. (Notice that we are talking of a single roll of a pair of dice here. We are not talking of rolling a pair of dice and getting "odd sum" and then rolling *again* and asking about the probability of "getting doubles.")

b. Suppose event A is "gets 1 on first die" and event B is "gets sum of seven." Then $P(A) = \frac{1}{6}$. If the sum of the dice is already known to be 7, then there are six ways this could happen: 1,6; 2,5; 3,4; 4,3; 5,2; 6,1. Again only $\frac{1}{6}$ of these has "1" on first die. So, $P(\textit{1 on first die}|\textit{sum is already known to be 7}) = \frac{1}{6}$. So, $P(A) = P(A|B)$.

c. Answer left to students.

Lesson 3, page 491, Organizing Task 4

a. The theoretical distribution was made in class. See page 487. The beginning of the table is given below. Since this is a theoretical situation, you can have "parts" of a person, and the wait time could be any number from 1, 2, 3, … . If you start with 36 people and a probability of $\frac{1}{6}$ for "doubles," then $\frac{1}{6}(36)$ will be released on first throw.

Number of Rolls to Get Doubles	Expected Number of Persons Released	Expected Number of Persons Still in Jail
1	6 ($\frac{1}{6}$ of 36)	30
2	5 ($\frac{1}{6}$ of 30)	25
3	$\frac{25}{6}$ or 4.17	$20\frac{5}{6}$
4	$\frac{125}{6}$ or 3.47	etc.

Students should make a graph with number of rolls to get doubles on the horizontal axis and expected number of persons released on the vertical axis.

b. Students should notice that they released $\frac{1}{6}$ and retained $\frac{5}{6}$ each time. On each succeeding number of rolls, to get the last two columns they multiplied the previous number by $\frac{5}{6}$. If they do not notice the exponential decay pattern in the table, they should notice the familiar exponential shape of the graph. To write the exponential equation in the $y = ab^x$ form, you need to know the y value for $x = 0$ rolls. You could either work back up the table to get $\left(\frac{6}{5}\right)(6) = 7.2$ or use calculators to find an **Expreg** equation to fit the data. Either way, you get $y = 7.2\left(\frac{5}{6}\right)^x$, where $x = 1, 2, 3, \ldots$.

c. Answer left to students.

Lesson 3, page 503, Modeling Task 2

a. The probability of getting the tiger sticker on the first try is 0.25. The probability you don't get the tiger on the first try is 0.75. If you get the tiger on the second try, then the outcome is "not tiger, and then tiger." The probability of "not tiger and then tiger" is *P*(*not tiger, and then tiger*) = *P*(*not tiger*)*P*(*tiger*) = (0.75)(0.25). If you get the tiger on the third try, then the outcome is "not tiger, not tiger, and then tiger" and the probability of this is $(0.75)^2(0.25)$, and so on. The answer is $P(x) = (0.75)^{x-1}(0.25)$, where x is the number of purchases needed to get a tiger sticker.

b. Answer left to students.

Lesson 3, page 505, Organizing Task 1

a. If p is the probability that an event will occur, then the probability that it will occur on the first trial is p.

Number of Trials, *x*	**Probability, *P*(*x*)**
1	p

b. $1 - p$

c. $P(\text{non-event and then event}) = (1 - p)(p)$

d. Answer left to students.

e. Answer left to students.

f. Answer left to students.

Students studied exponential relationships in Course 1, Unit 6. Therefore, the idea of repeatedly multiplying by a factor (in this case a decay factor) should be familiar to them in the form $y = ab^x$. The form $y = ab^{x-1}$ is also an exponential equation since ab^{x-1} means a constant "a" times a constant "b" raised to a power "x – 1."

These kinds of connections among important pieces of mathematics, from different fields, in this case algebra and probability, are deliberately sought and fostered in the curriculum. Students are encouraged to make and see connections, which in turn leads to better retention of ideas than would occur if ideas were to be developed in isolation. In this case, the algebraic relationship makes the pattern of probabilities in the distribution clearer.

